CPA

Aims of the session

- *To explain the rationale behind the promotion of the CPA approach.*
- *To consider implications for the whole school for this approach.*
- *To look at how resources can be used across the year groups.*

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Warm up

You have 4 bags of numbers.

One filled with the number 1, one filled with the number 3, one filled with number 5 and one filled with number 9.

Take any ten of these numbers to make a total of 37.

So what is CPA?

Jerome Bruner

Children need to experience a mix of three different modes of learning:

Enactive, Iconic and Symbolic.

"learning begins with an action - touching, feeling, and manipulating"

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Whilst Bruner's work referred to distinct stages in child development these broad principles have been adopted by the Education Ministry in Singapore and referred to as Concrete–Pictorial–Abstract [CPA] which are not considered sequential.

(https://repository.nie.edu.sg/bitstream/10497/18889/1/TME-16-1-1.pdf)

CONCRETE	PICTORIAL	ABSTRACT
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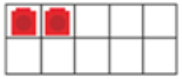
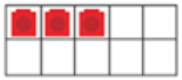
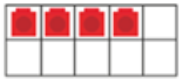
		2
		3
		4

C P A

Enactive

Concrete

Iconic

Pictorial

$5 = 3 + 2$

Symbolic

Abstract

Manipulatives and representations

MANIPULATIVES	REPRESENTATIONS

Manipulatives and representations

MANIPULATIVES	REPRESENTATIONS
A manipulative is an object that children or practitioners can interact with and move to represent mathematical ideas. Manipulatives could include everyday objects such as pine cones, buttons, and small toys as well as resources like interlocking cubes, Cuisenaire® rods, Dienes blocks, and building blocks.	A 'representation' refers to a particular form in which mathematics is presented. Representations include informal drawings, mathematical symbols, and more formal diagrams, such as a number line or graph.

There are 7 jelly beans and 5 fruit pastels, how many sweets altogether?

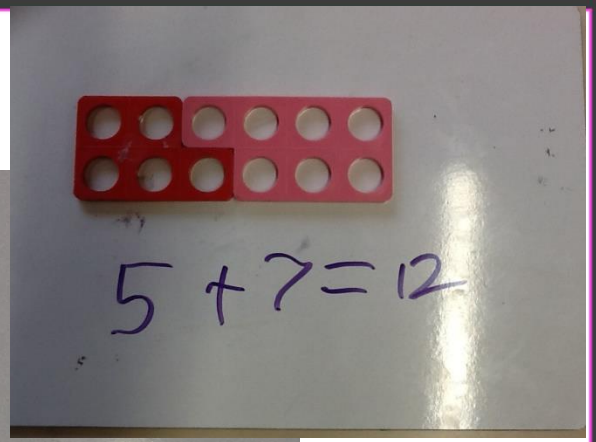
"Show me"

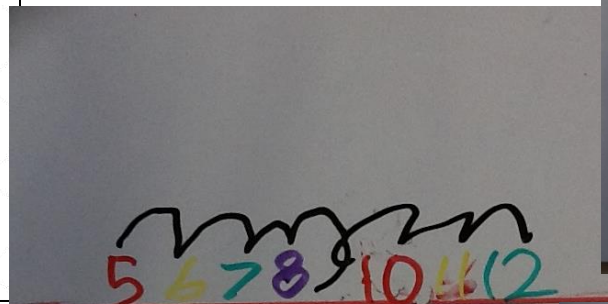
"Prove it!"

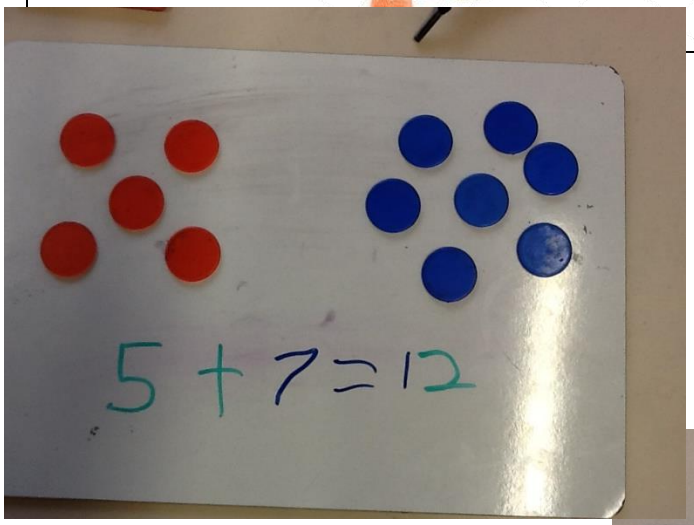
This is how children in Year 1 and 2 approached the problem …

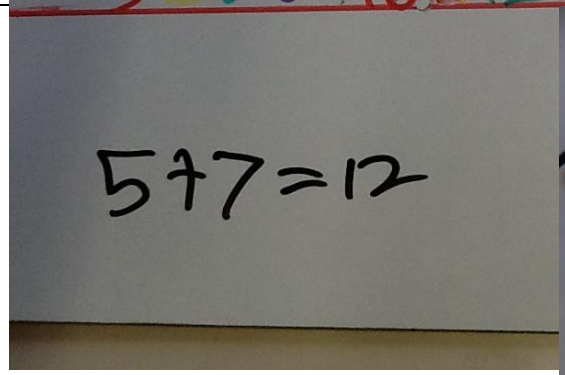
There are 7 jelly beans and 5 fruit pastels, how many sweets altogether?

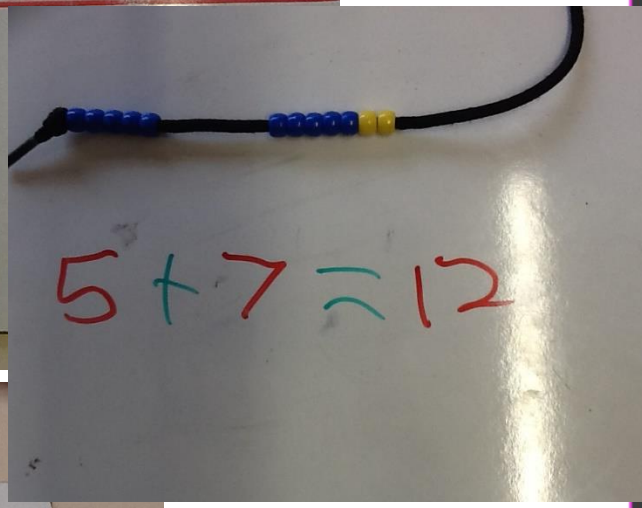
"Show me"
"Prove it!"

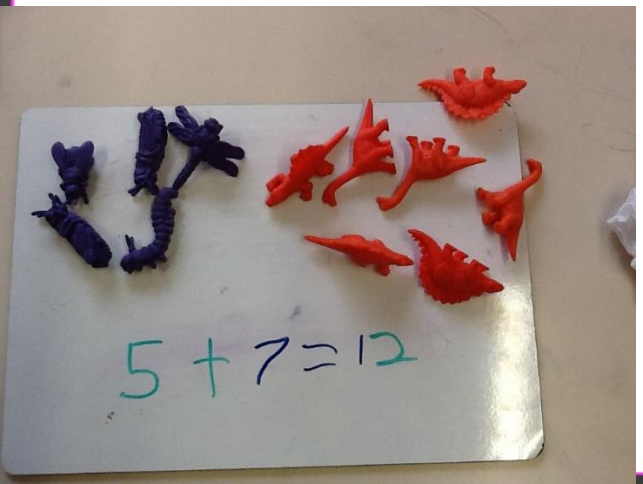


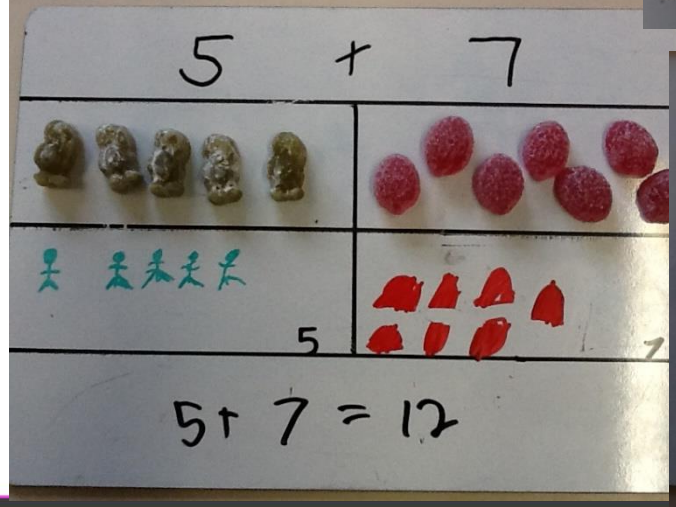


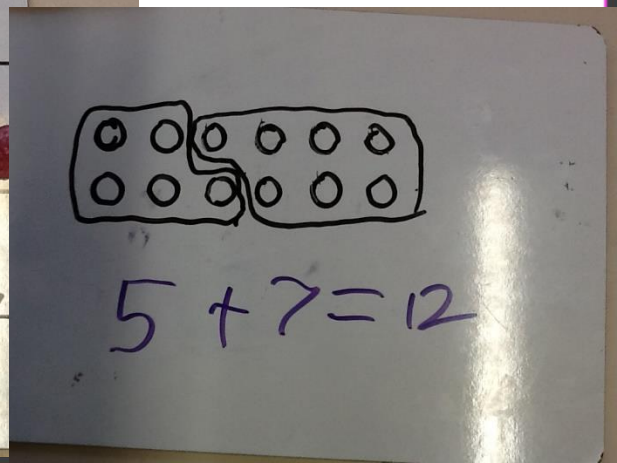












CPA in the Curriculum

The final version of National Curriculum for mathematics aims to ensure all pupils:

- become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and are able to recall and apply their knowledge rapidly and accurately to problems
- **reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

A mathematical concept can be thought of as a network of connections between symbols, language, concrete experiences and pictures

Derek Haylock and Anne Cockburn 2008

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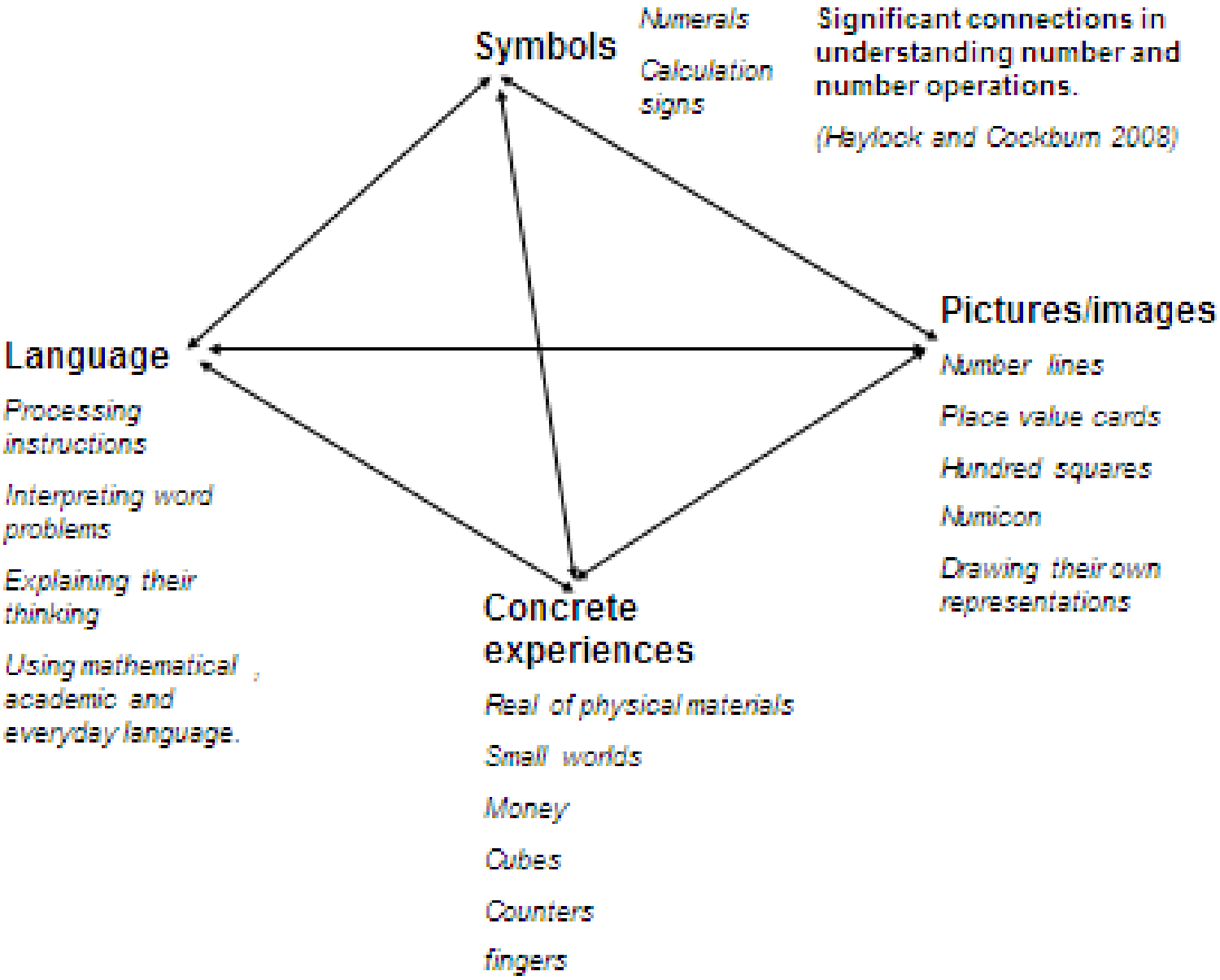
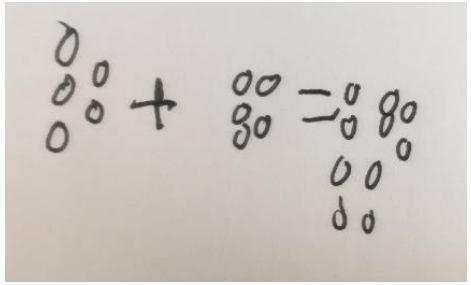
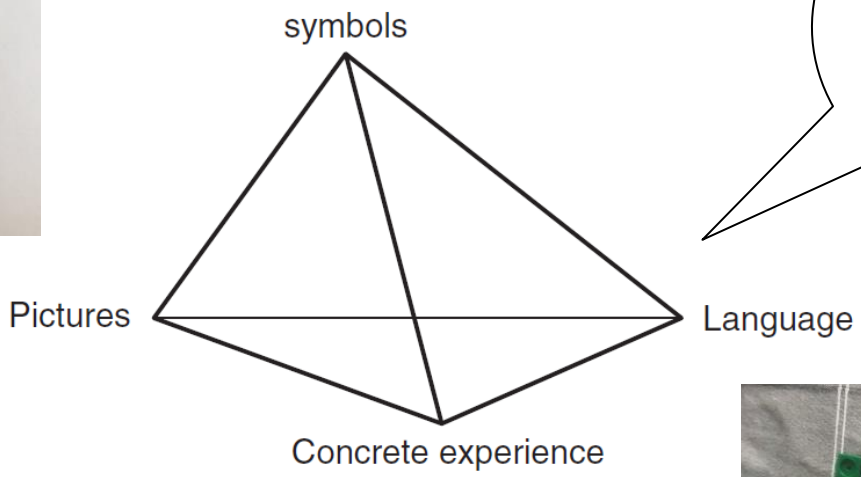


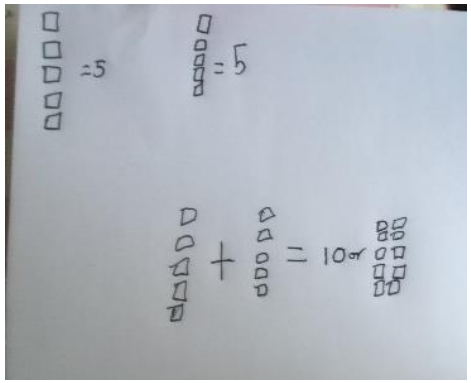
Derek Haylock – making connections

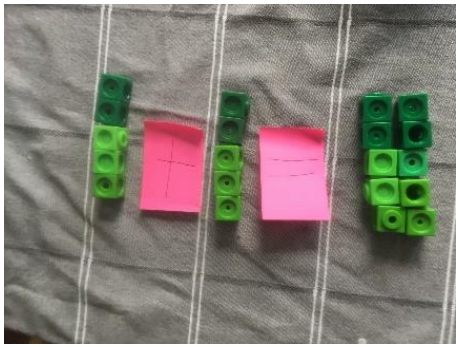


5+5=10



If you have 5 then doubling means the same again so you have another 5





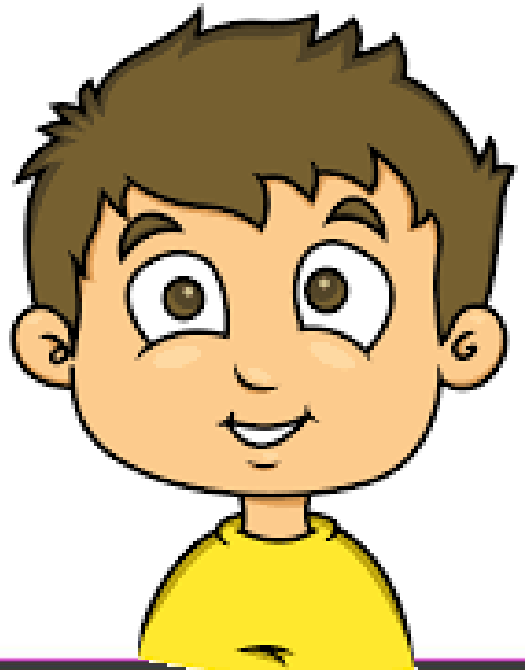
Haylock, D. and Thangata, F. (2007) 'Making connections.' *In Key concepts in teaching primary mathematics.* London: SAGE Publications

I can …

Build it

Draw it

Write it



… and explain it.

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Piaget

Piaget's work on Schemas underlined this need for variation to establish facts.

Every experience builds on the picture.

Leads to depth of learning and mastery.

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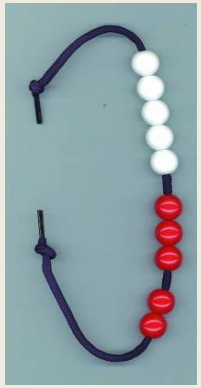
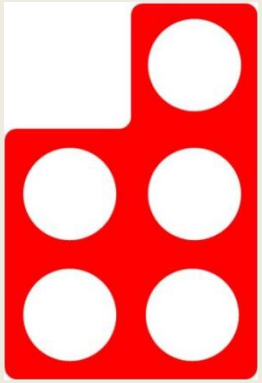
If you have provided one view, that is the only way the students will see it.

My experience is the limit of my world.





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	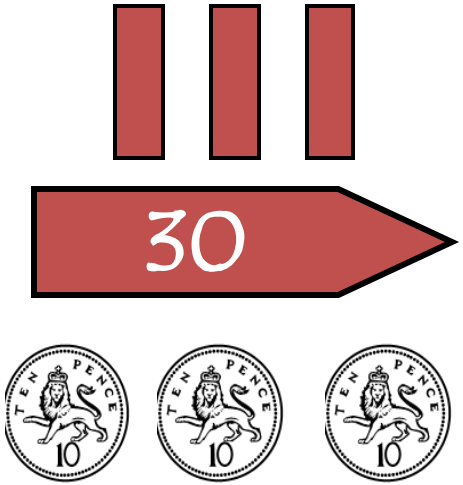	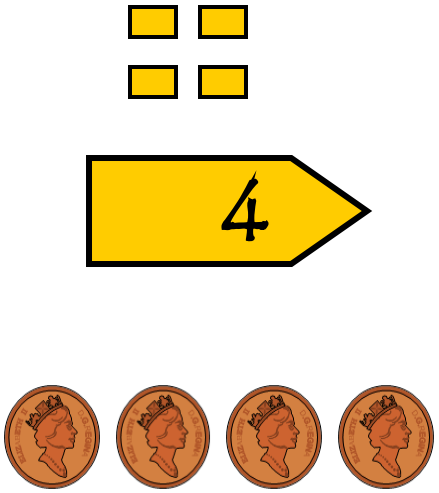

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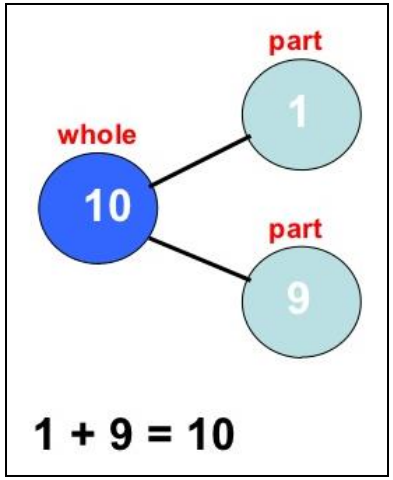

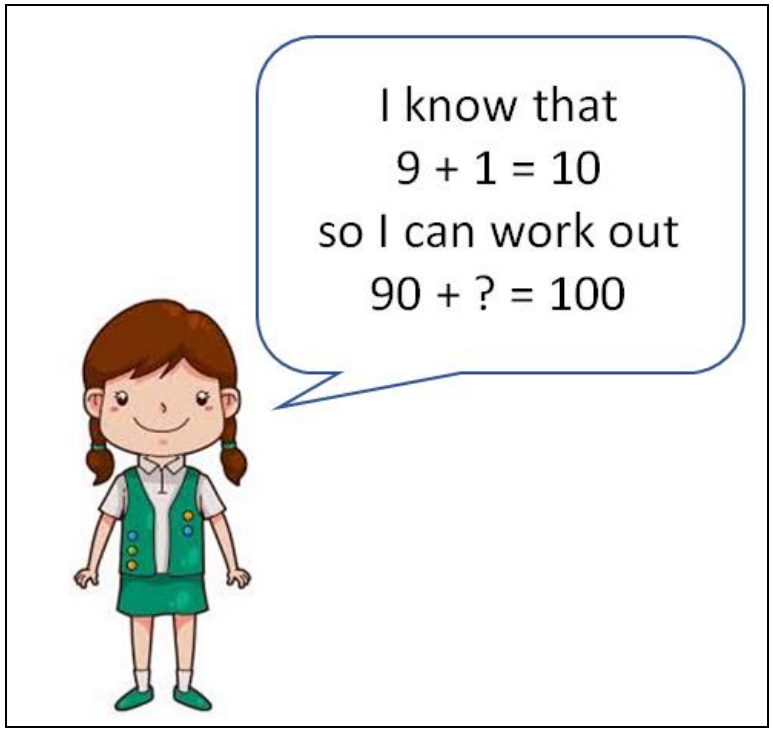

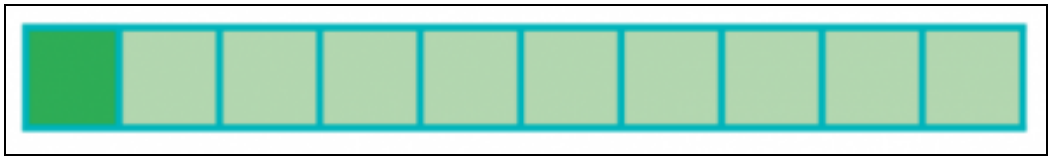

They will be “stuck” in that representation.

Freddie's Video – place value

Watch the video of Freddie

What do we know about Freddie's understanding of place value?

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If you have provided one view, that is the only way the students will see it.

My language is the limit of my world.

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Children can only explain their reasoning if they have the words.

combine
double check
–
subtract
inverse
addition
=
+
add
calculate
represent
calculation
subtraction

Model the full range of language – maths talk.

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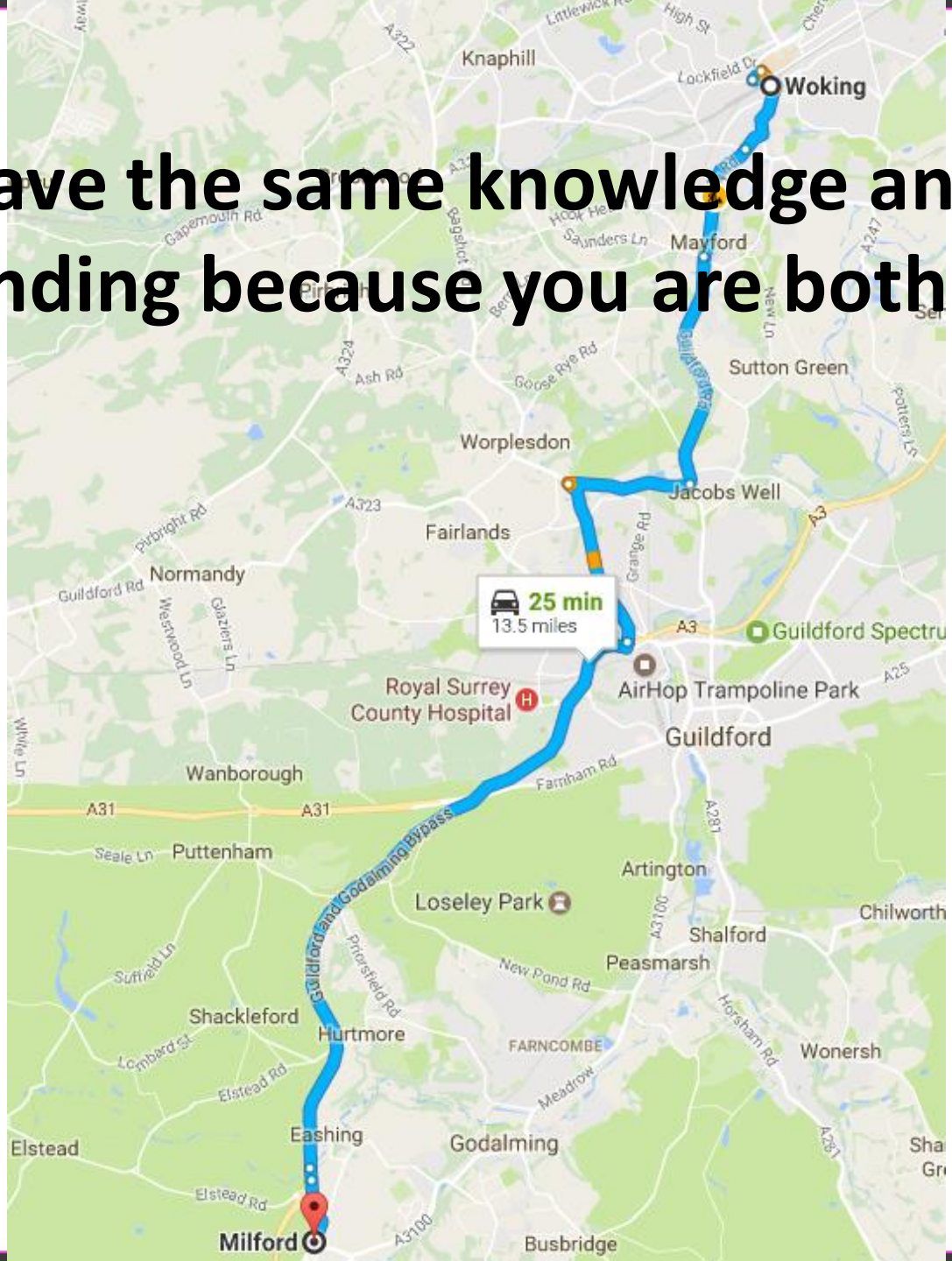
Do you have the same knowledge and understanding because you are both here?



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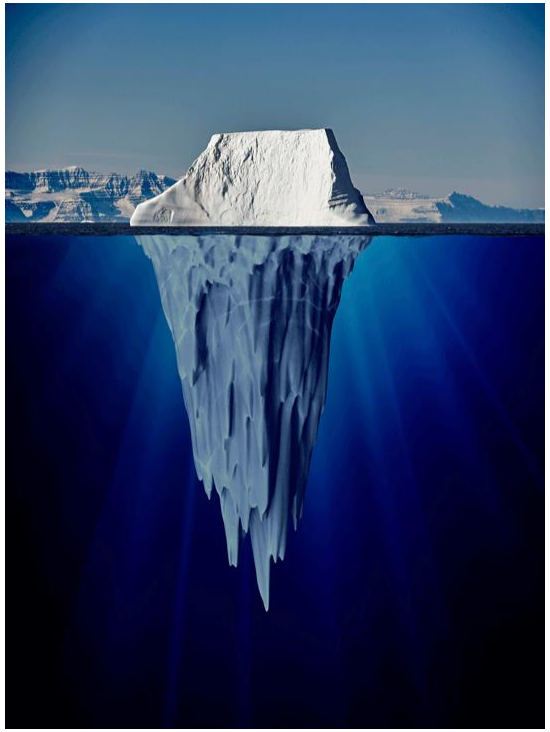
What do they know?

Both of these look the same from the surface.



Unless you ask the right question you won’t know until they move on to something new.

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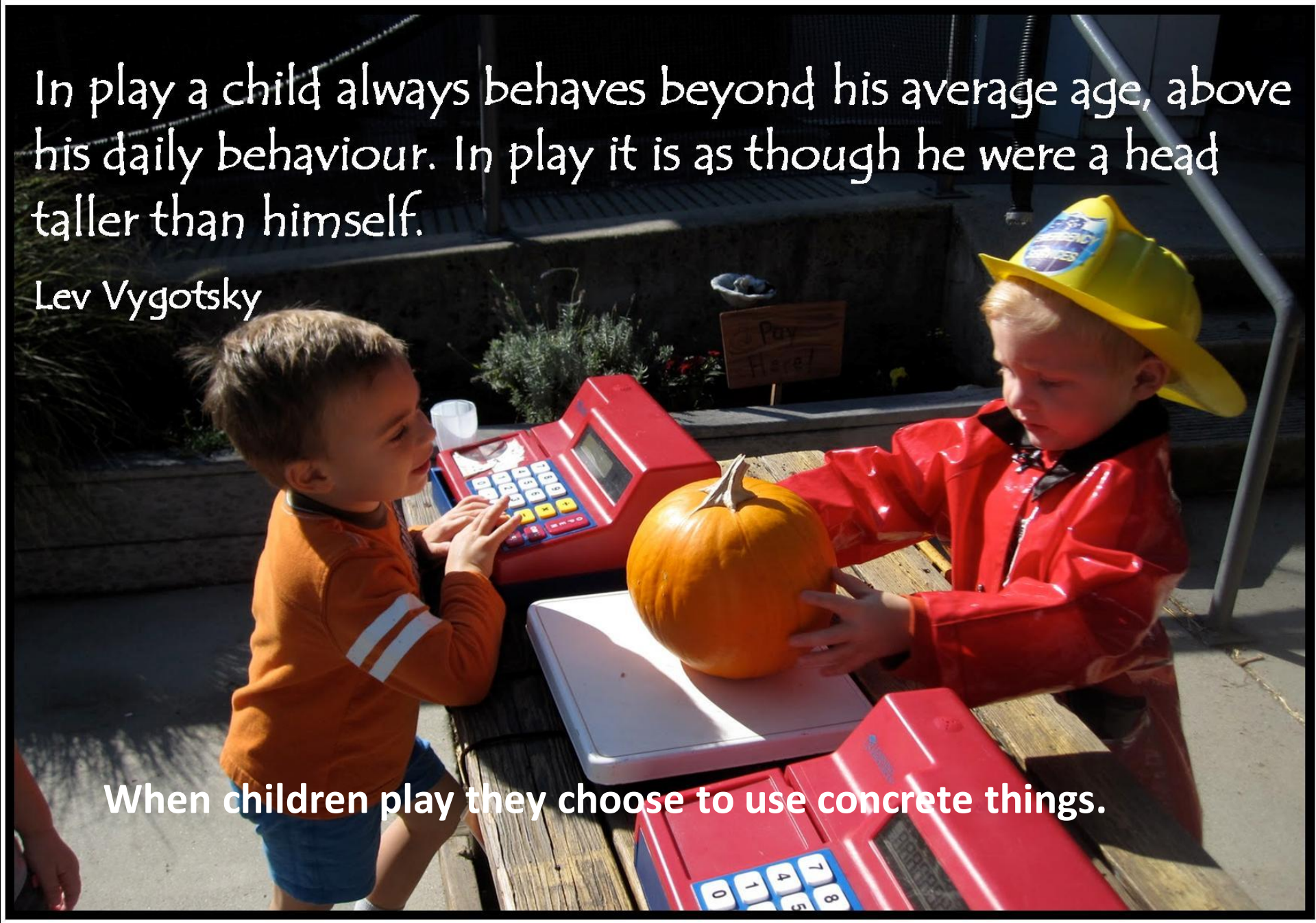
In play a child always behaves beyond his average age, above his daily behaviour. In play it is as though he were a head taller than himself.
Lev Vygotsky
Pay Here!
When children play they choose to use concrete things.

Here are some of our maths toys …

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Magic math
0
1
2
4
5
6
7
8
9
10

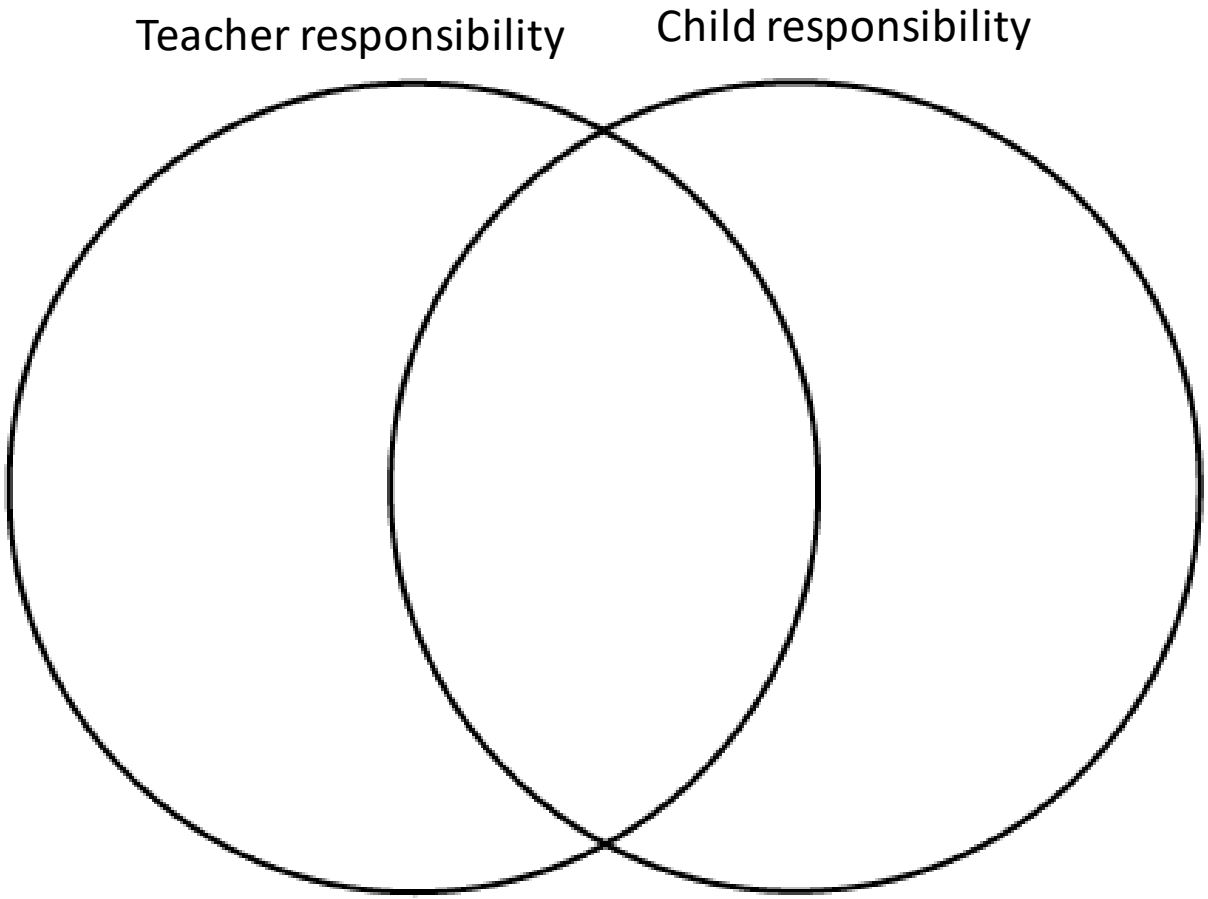

0 1 2 3 4 5 6 7 8 9 10

Organisation & Displays


MATHS
This week we are learning about....
Angles and Turns
Angles + turns
1/4 turn = 1 right angle
1/2 turn = 2 right angles
whole turn = 4 right angles
This week's mathematical vocabulary
Right angle
degree
Whole turn
Half turn
quarter turn
Odd Numbers
1 3 5 7 9
Even Numbers
2 4 6 8 10
Round up or down?
under 5
5+
GOLD
SILVER
BRONZE
Number bonds to 100
56+L = 100
+4
+40
56
60
100
right angle
90°
0°-90°
acute angle
90°-180°
obtuse angle
Column Addition with Partitioning.
HTU
241
+126
367
H T U
200 + 40 + 1
100 + 20 + 6
300 + 60 + 7

Organisation of resources

Expectations and responsibility



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Manipulatives and Representations throughout the school

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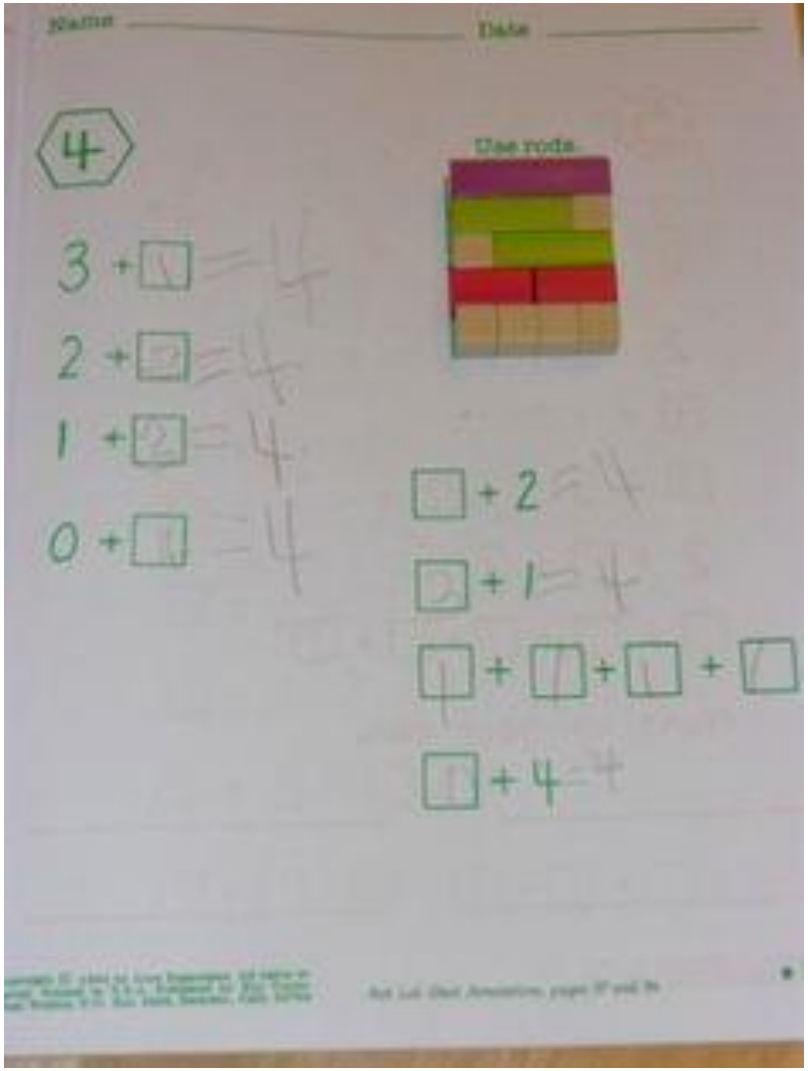
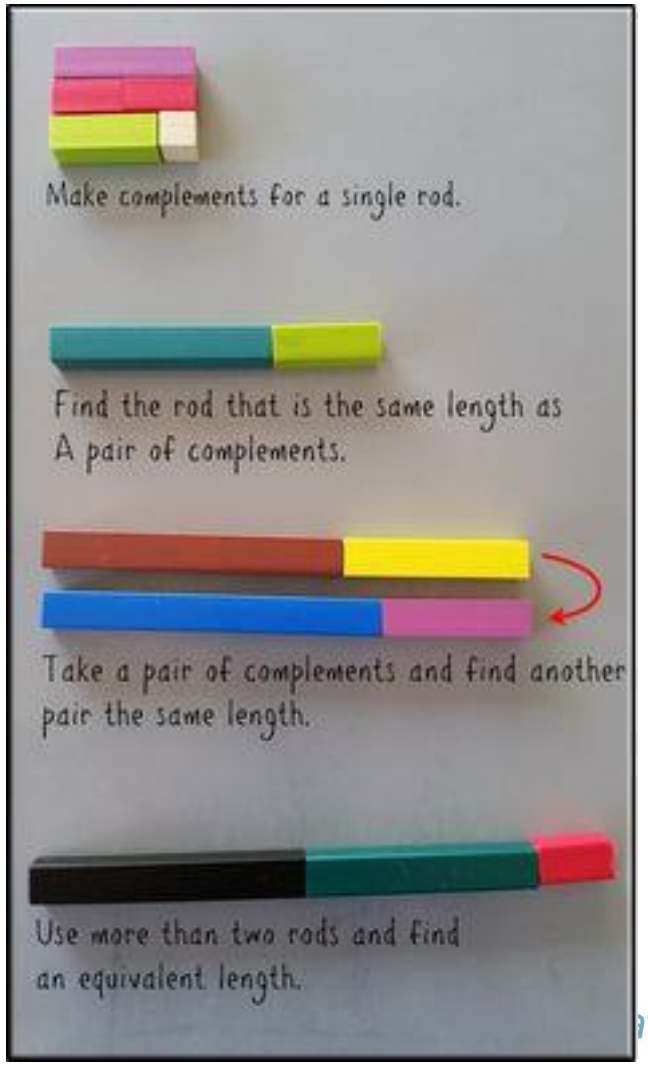
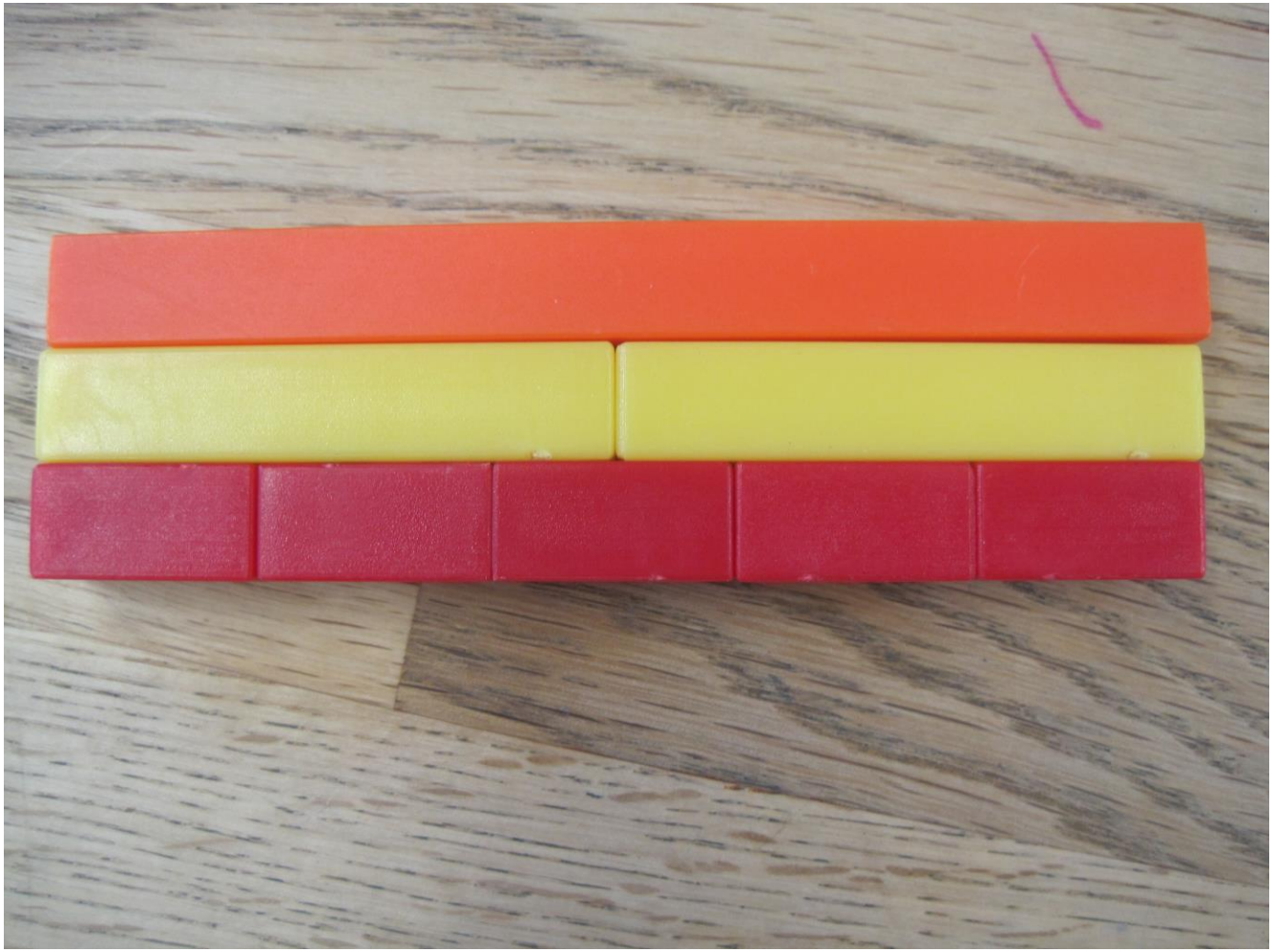
Cuisenaire for missing numbers





Cuisenaire for problem solving: division



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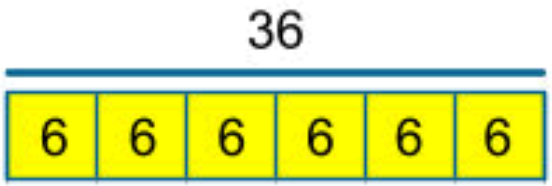
Cuisenaire for problem solving: division

Division

When using the bar model for division it is the image of sharing rather than grouping which is highlighted in this representation.

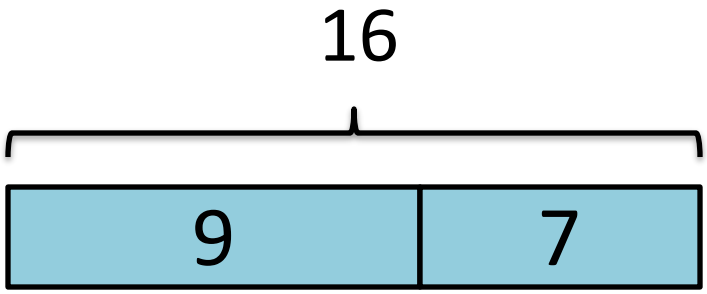
Mr Smith had a piece of wood that measured 36 cm.
He cut it into 6 equal pieces.
How long was each piece?

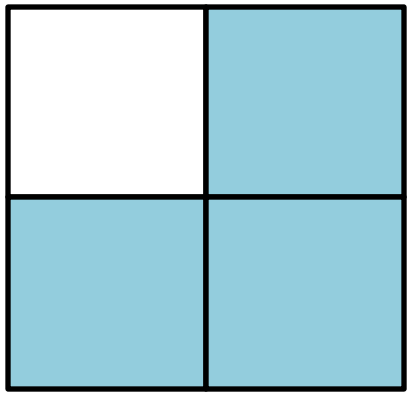


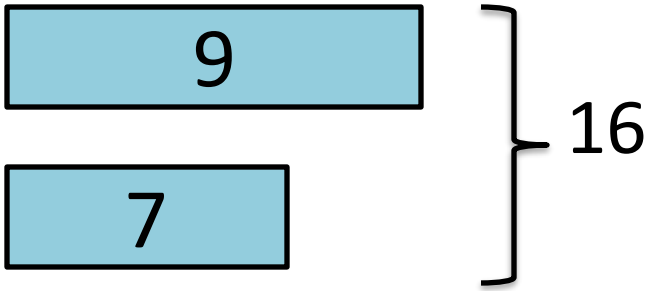
36 ÷ 6 = 6
Each piece is 6 cm

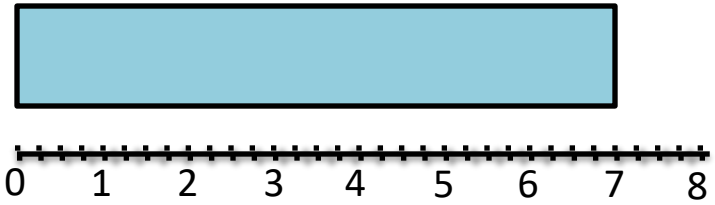
What is the Bar Model?

The bar model is a representation of the structure of a maths problem using rectangles. It is designed to help children visualise maths problems rather than do the maths for them.









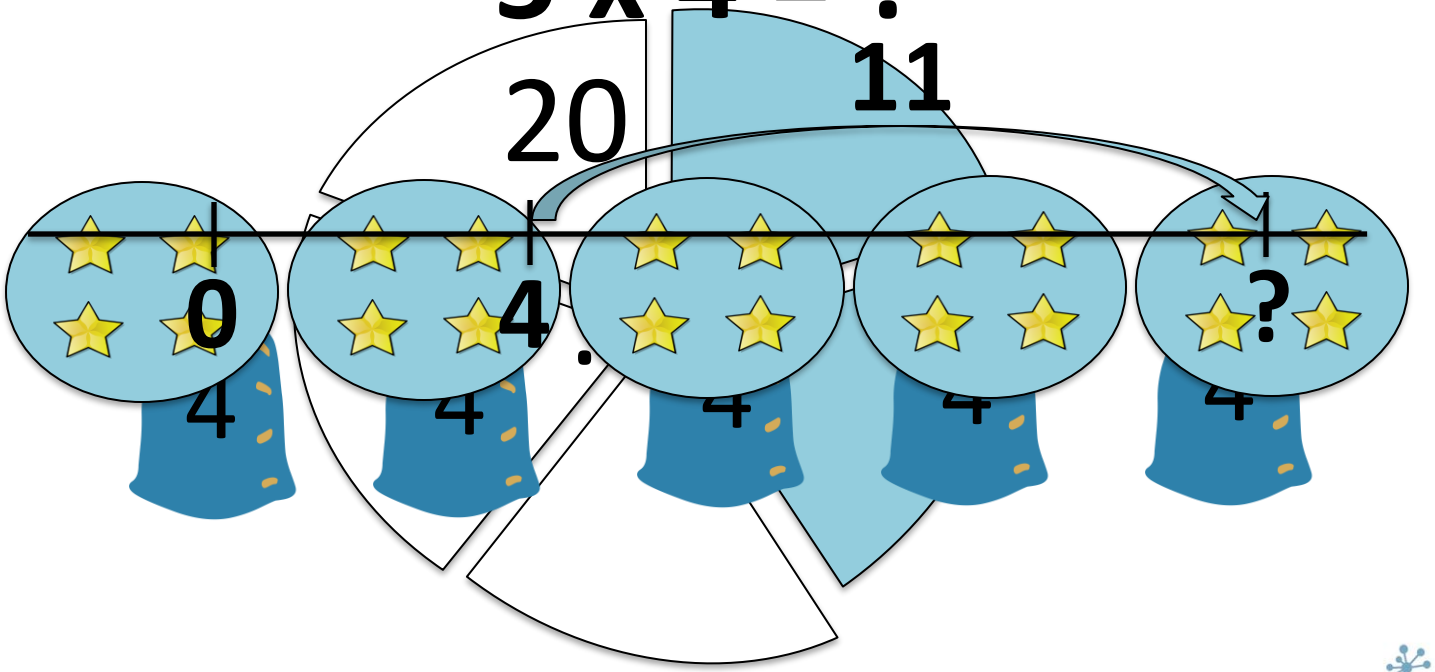
A Consistent Picture?

$\frac{2}{5}$ of 20 = ?

4 + 11 = ?

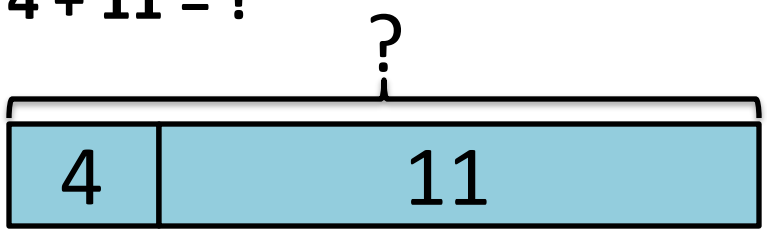
Share 20 in the ratio 2:3

5 x 4 = ?

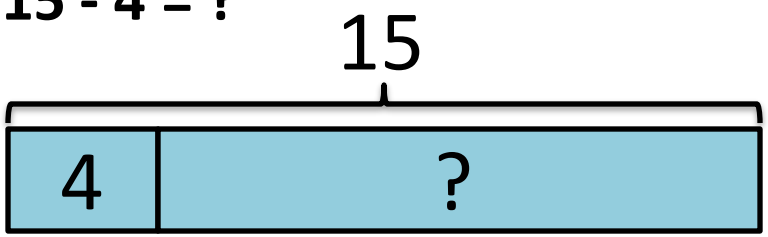


A Consistent Picture!

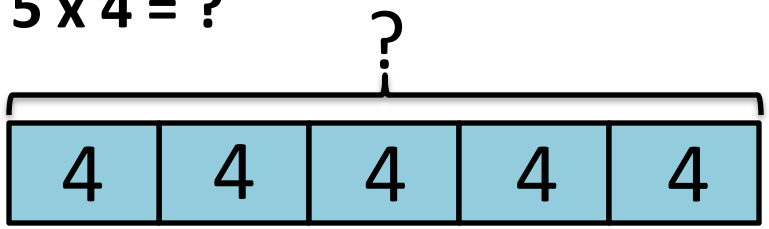
4 + 11 = ?



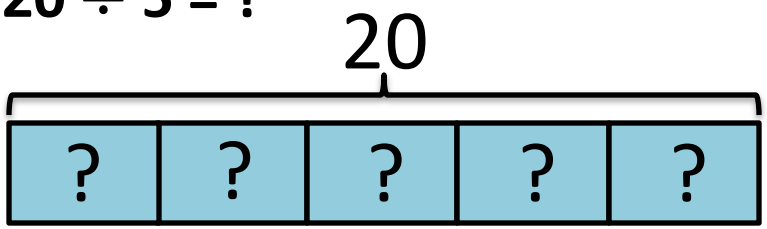
15 - 4 = ?



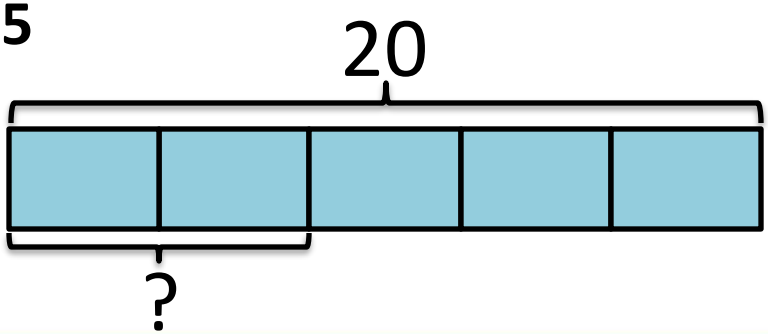
5 x 4 = ?



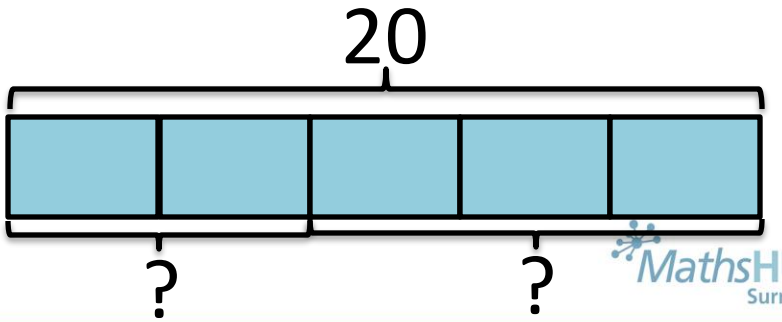
20 ÷ 5 = ?



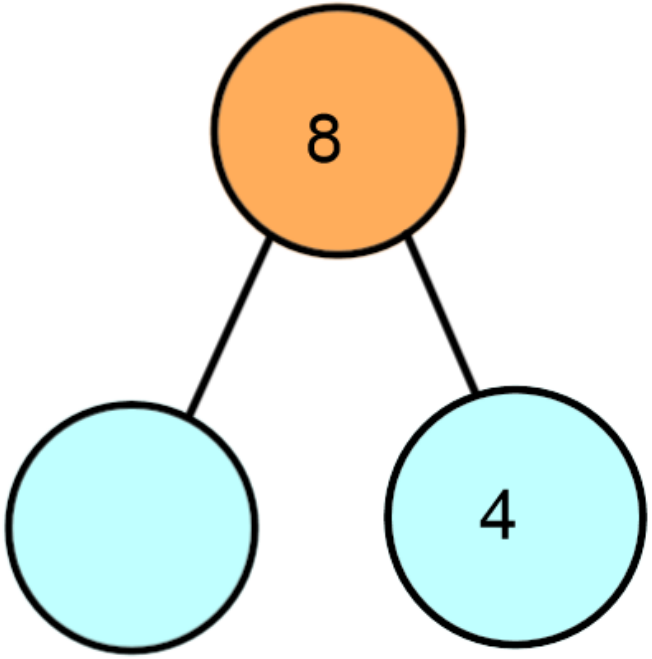
$\frac{2}{5}$ of 20 = ?

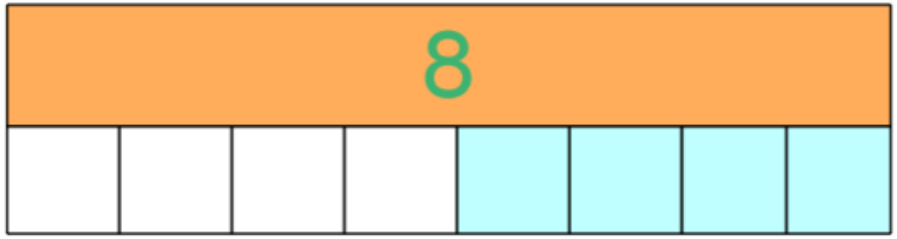


Share 20 in the ratio 2:3



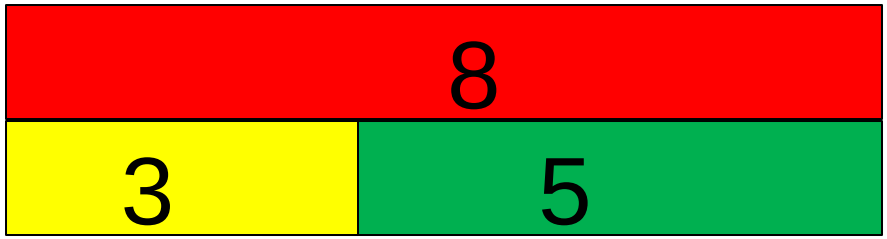
Part - part whole model





$\square + 4 = 8$

Part-Part-Whole Model



5 + 3 = 8

3 + 5 = 8

8 – 3 = 5

8 – 5 = 3

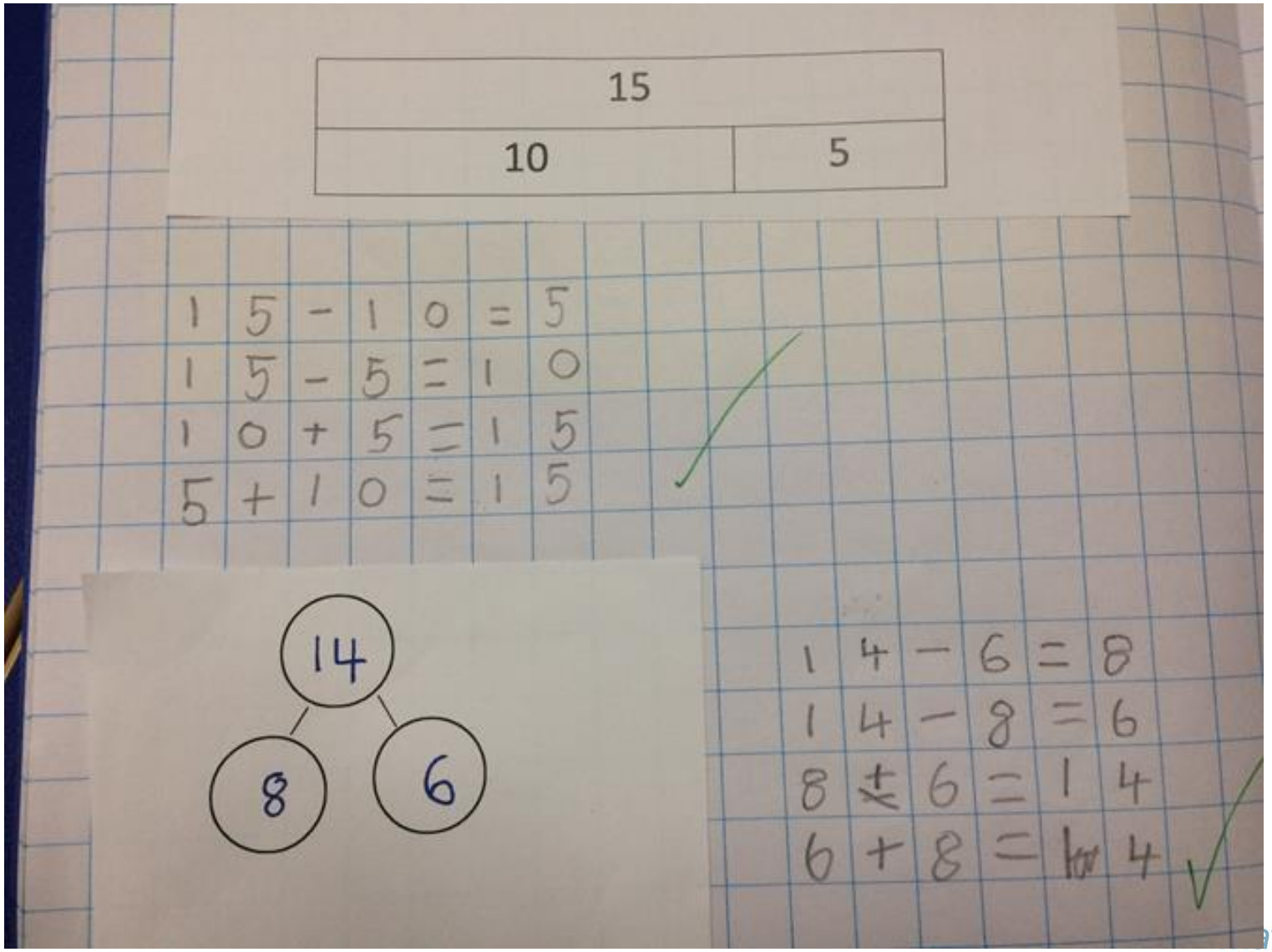
8 = 5 + 3

8 = 3 + 5

5 = 8 - 3

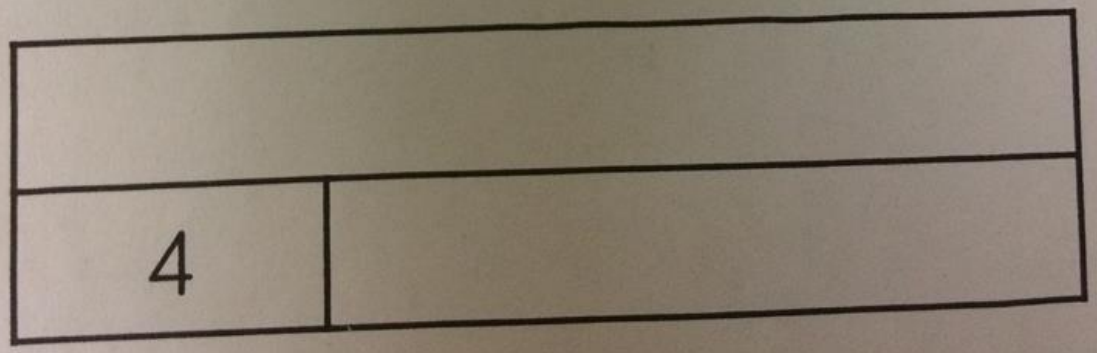
3 = 8 - 5

Year 2

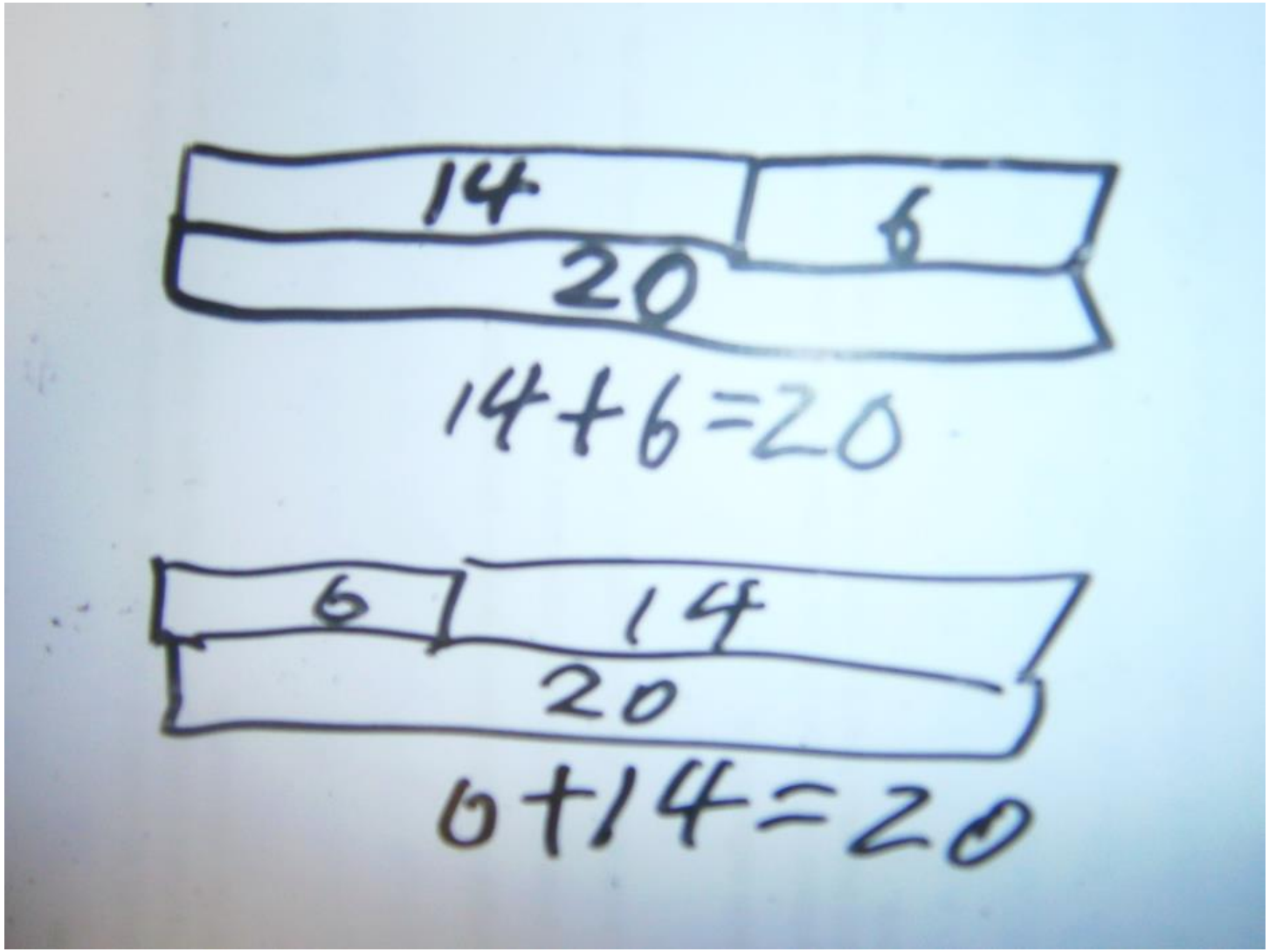


Year 2 – Greater Depth

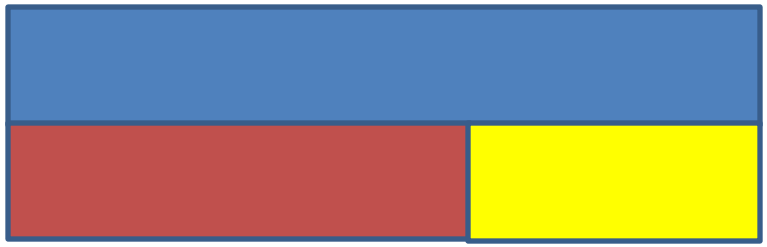
Here is an incomplete bar model.
The total is greater than 10 but less than 20.
What could the numbers be? How many different combinations can you find?



Practise drawing the bars



Addition and subtraction



The part-part-whole bar model can help children solve missing number problems throughout KS2

24 + ? = 56 ? + 76 = 168

? – 2 362 = 1 913 1.46 - ? = 0.79

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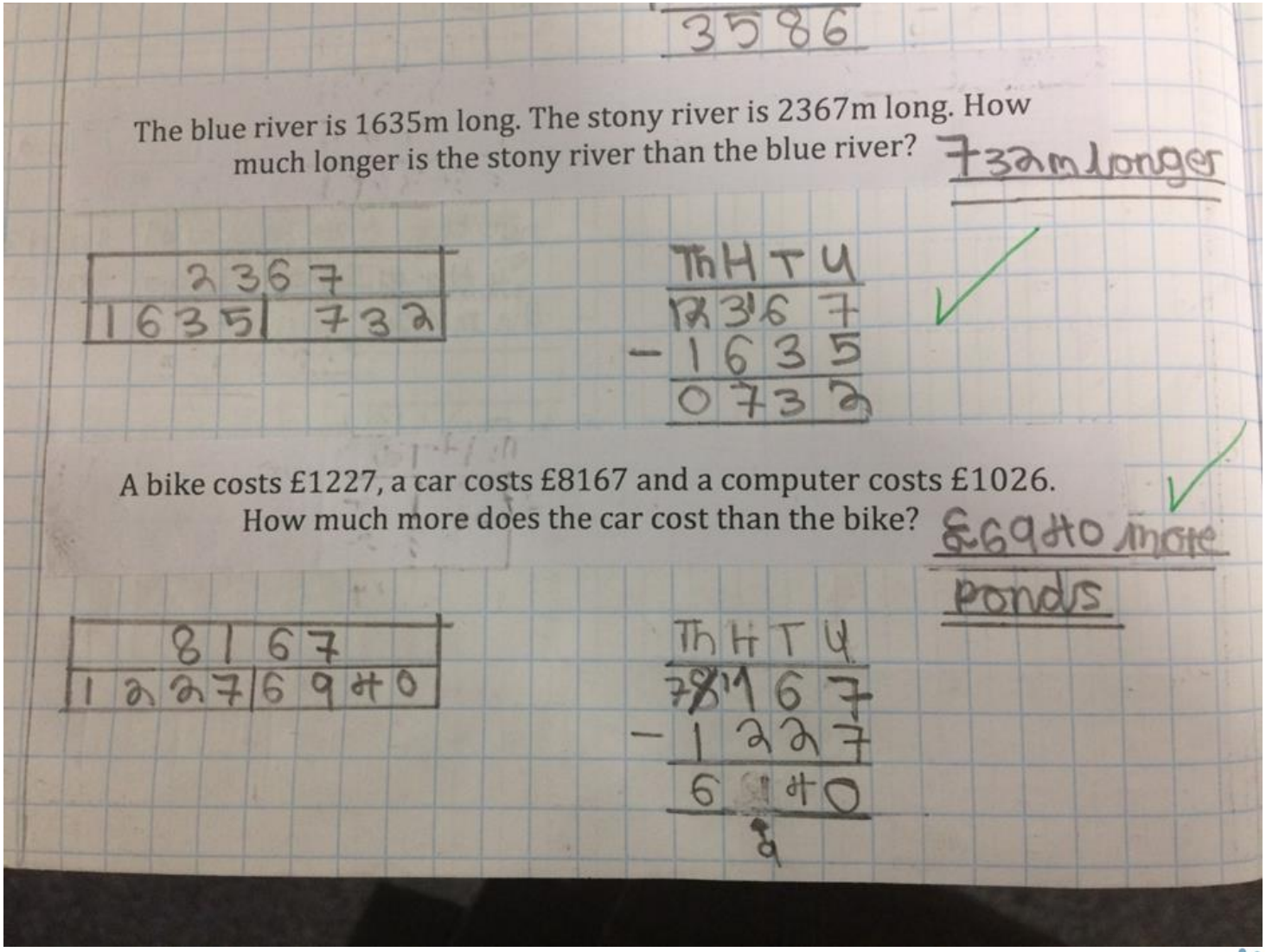
Year 5



3586

The blue river is 1635m long. The stony river is 2367m long. How much longer is the stony river than the blue river? 732m longer

2367	
1635	732

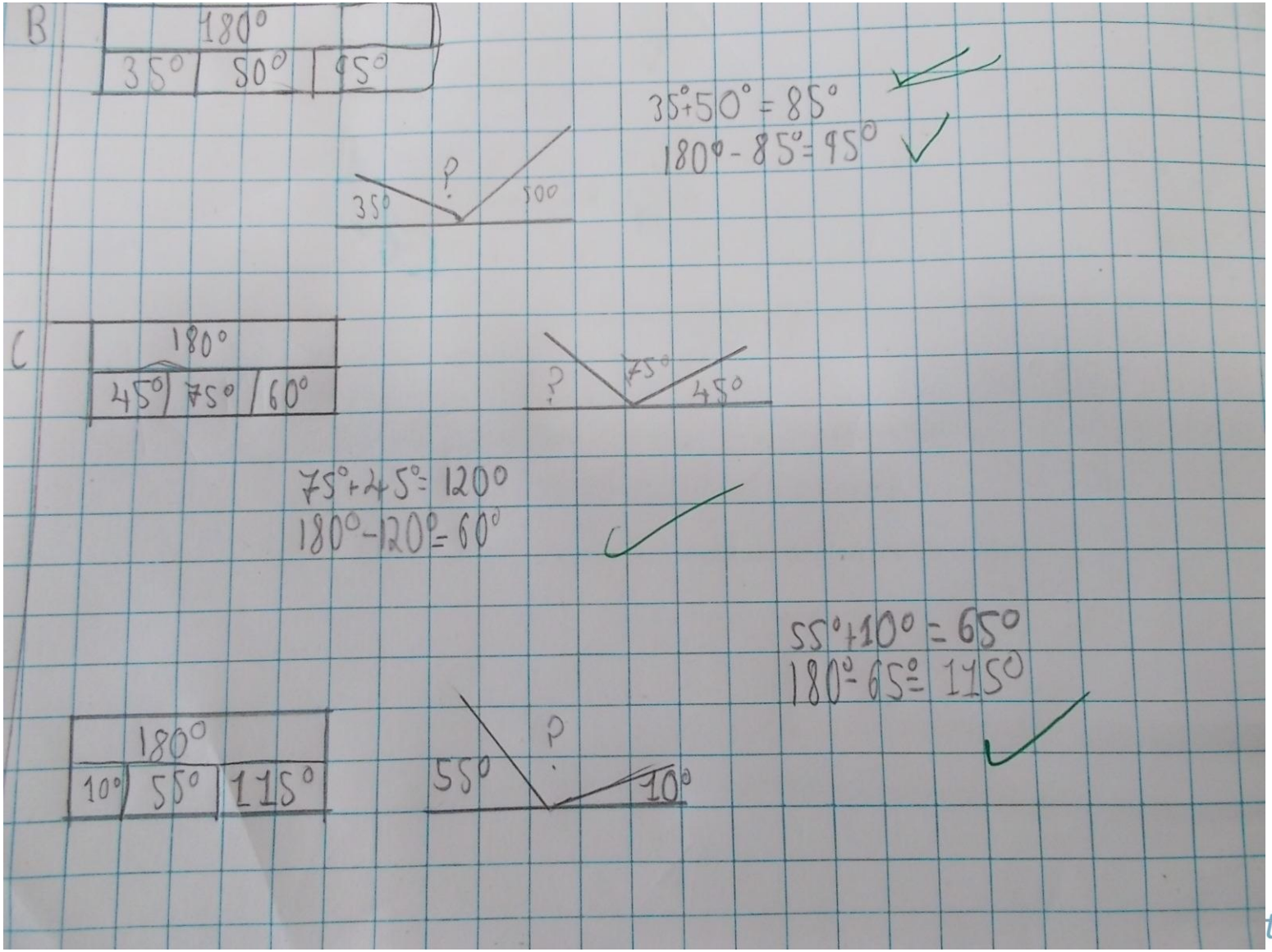
Th	H	T	U
1	2	3	7
− 1	6	3	5
0	7	3	2

A bike costs £1227, a car costs £8167 and a computer costs £1026. How much more does the car cost than the bike? £6940 more pounds

8167	
1227	6940

Th	H	T	U
7	8	1	6	7
− 1	2	2	7
6	9	4	0

Angles



Developing Depth/Simplicity/Clarity

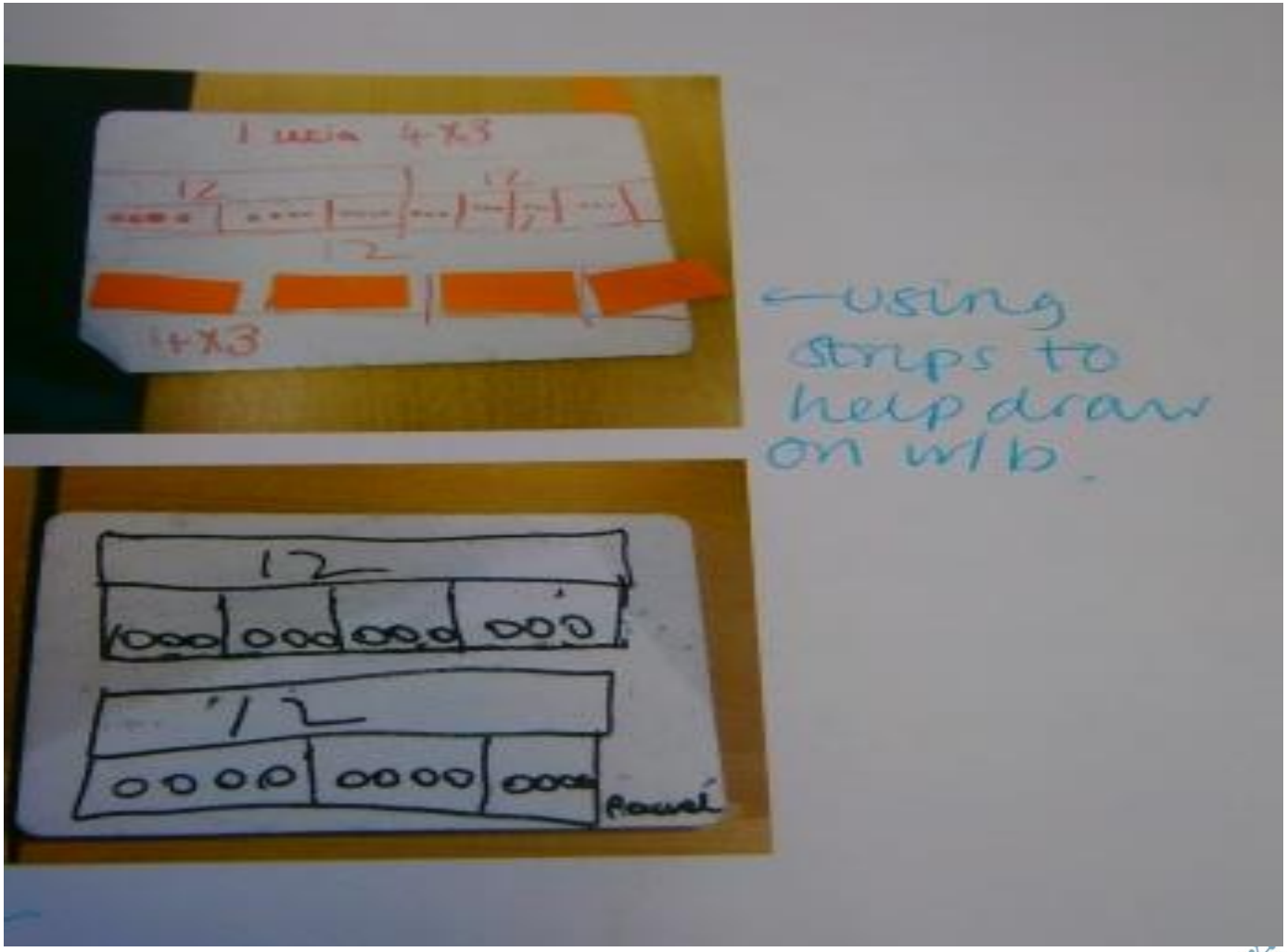
7	
2	5

0.7	
0.2	?

?	
200	500

a	
b	c

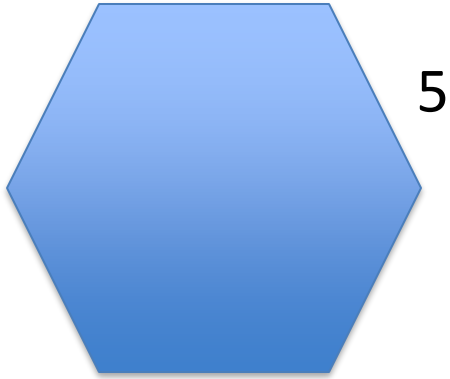
Multiplicative reasoning



← using strips to help draw on w/b.

Perimeter of Regular Shapes

- If one side of a regular hexagon measures 5cm, what is the perimeter of the shape?



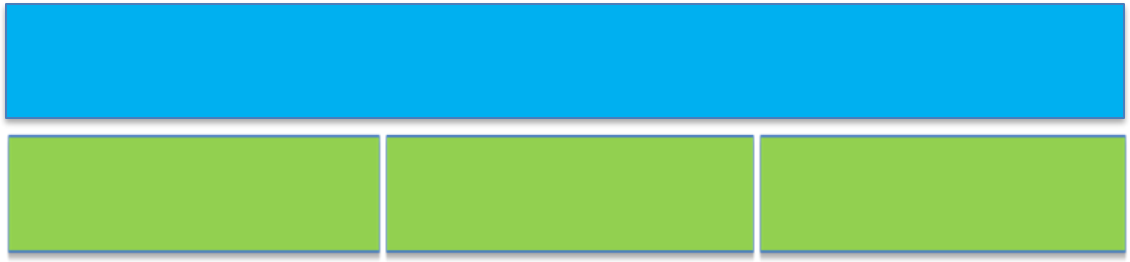
What could this model be showing?

Ratio of green to blue is 3:1

27 ÷ 3 = 9

3 x 4 = 12

3 x 1/3 = 1

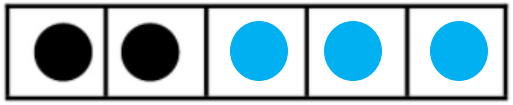


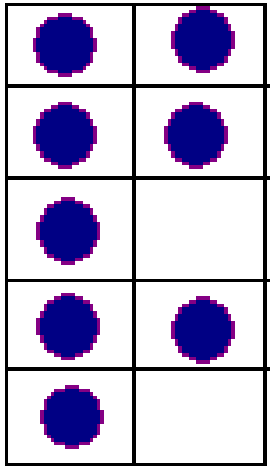
9 sweets shared between 3 bags

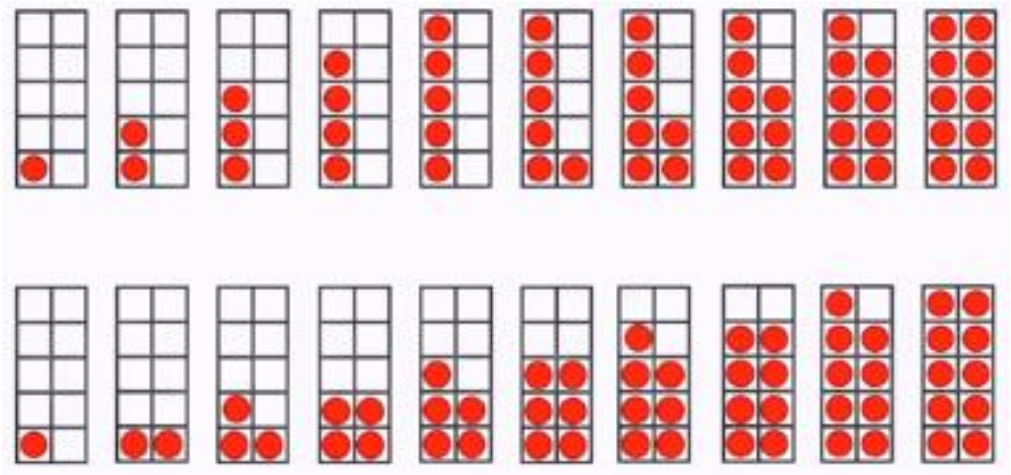
3 + 3 + 3 = 9

1/3 + 1/3 + 1/3 = 1

Tens Frames







Double sided counters

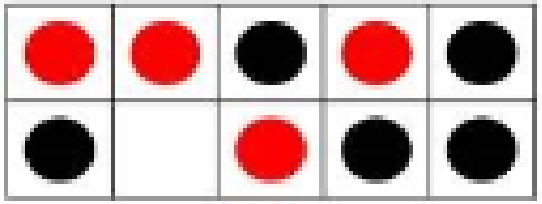
Can you make 7 on the ten frame?

Can you show me a different way to make 7?

Show me 2 more. What is the number now?

Show me 4 less than 10. What is the number?

Show me 2 more than 3. What is the number?

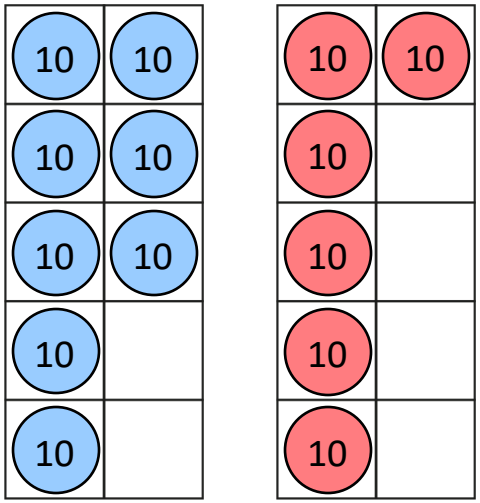


Can you show me a double fact?

Show me 3. How many more make 10?

Show me 7 + 5 on the ten frame. What is the answer?

Scaling number facts by 10



8 + 6

8 tens + 6 tens = _ . tens

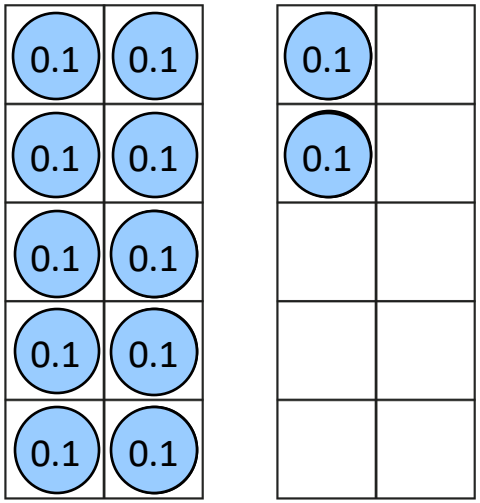
80 + 60

- Show the pupils the animated slide and record the calculations on a flipchart.
- What do you notice? What is the same about the two calculations? What is different?
- How can we use 8 + 6 to help us to find 80 + 60? How else can we say 14 tens?
- Repeat using other number pairs.

8 + 6 = 14

8 tens + 6 tens = 14 tens

Scaling number facts by 0.1 or 0.01

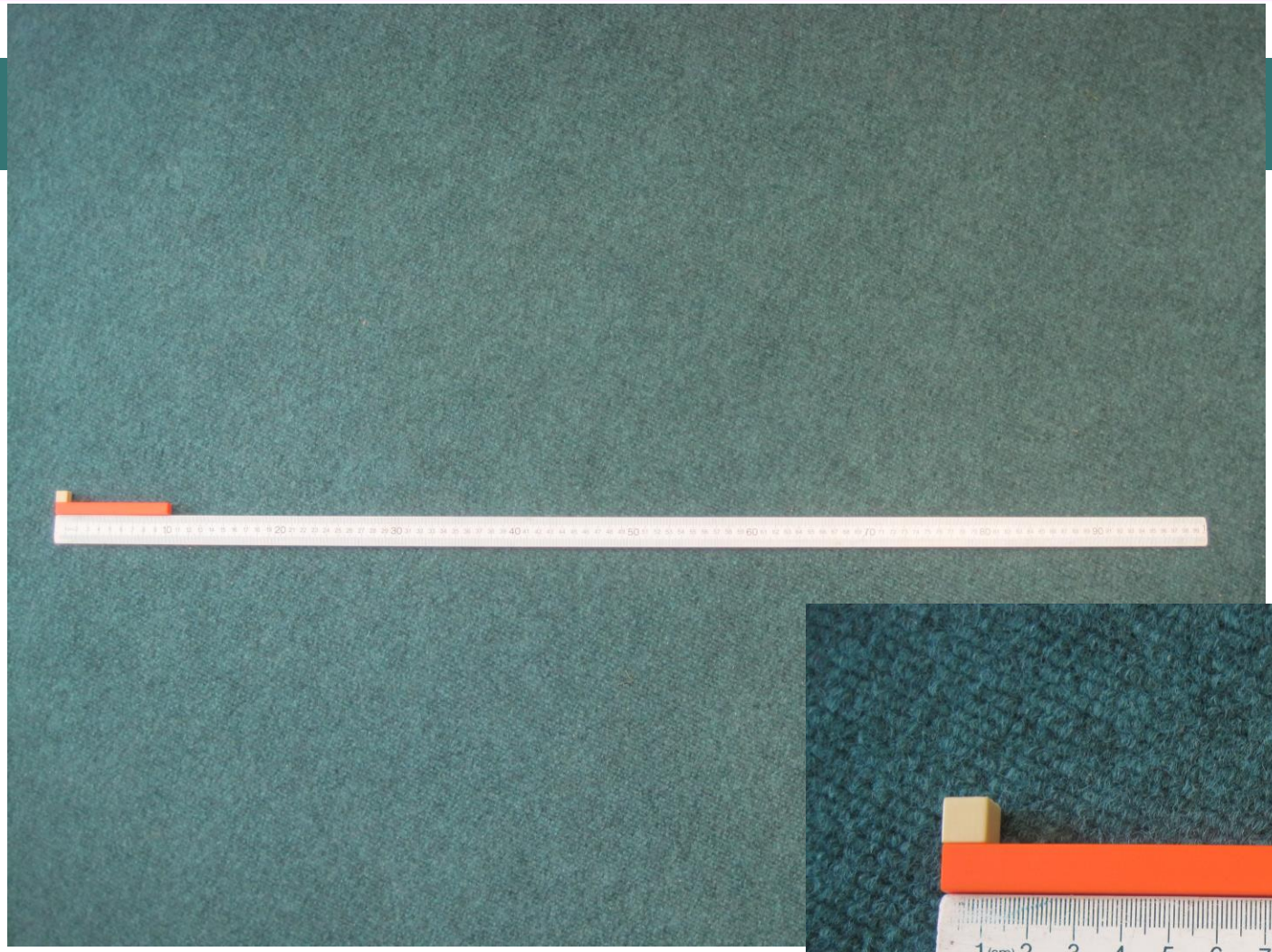


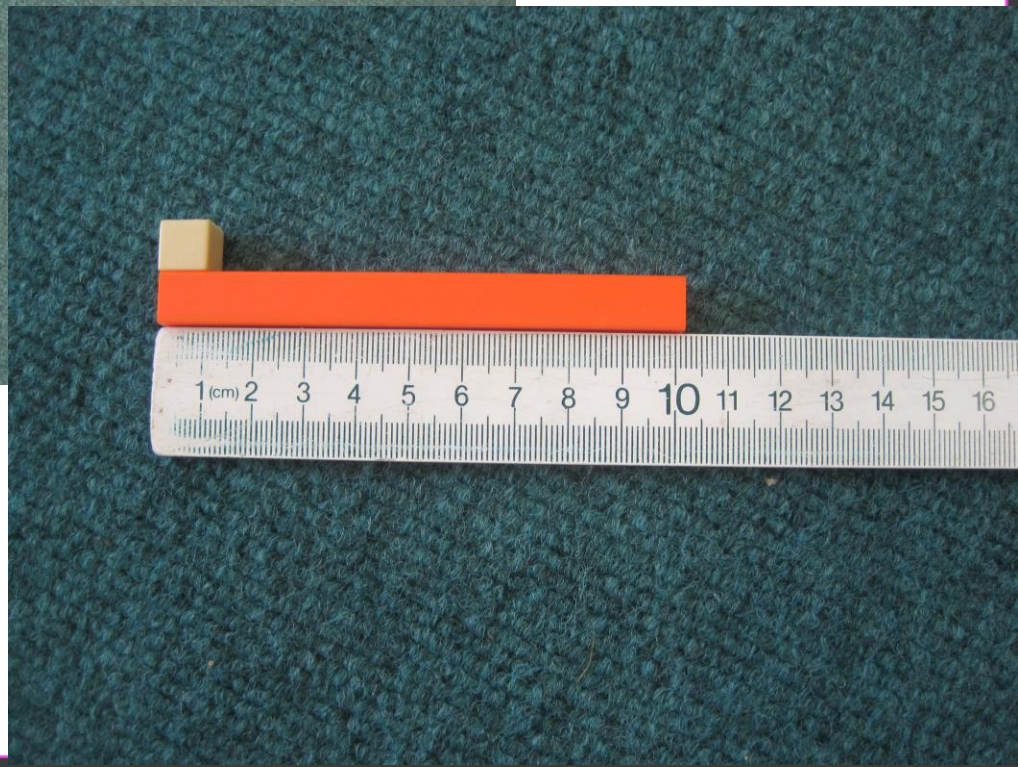
12 – 5

12 tenths – 5 tenths =

1.2 – 0.5

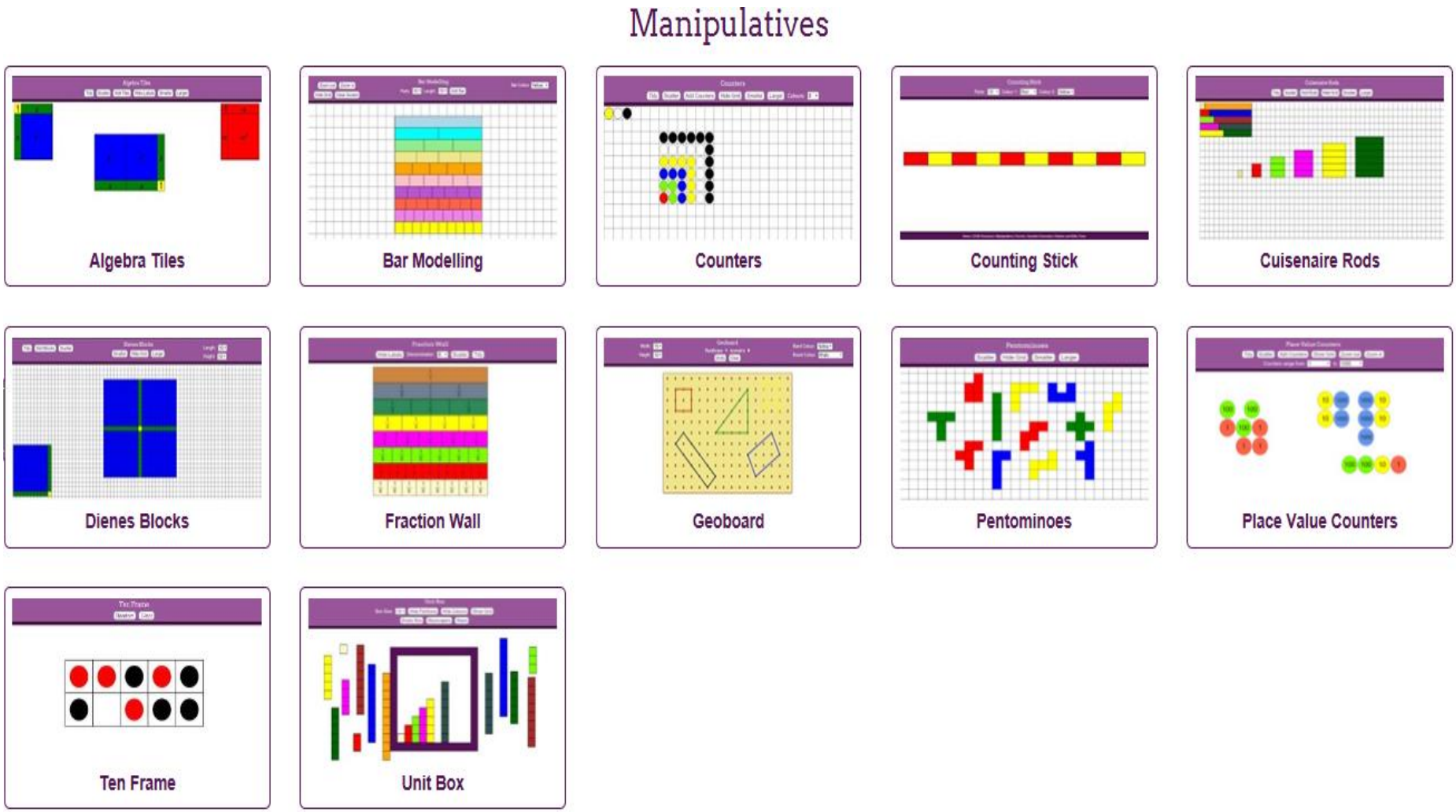
- Click to start the animation. Show me this representation using your tens frames and counters. How will you subtract 5? How many will you subtract first? If you subtract 2 first what are you left with? How many more do you need to subtract now?
- Lets try 12 tenths – 5 tenths now. What do you notice?
- Can you explain how you can use your knowledge of place value to solve this?
- Repeat for other similar number pairs bridging the ten each time.




1 (cm) 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16

Resources

Mathsbot.com - for pictorial representations



CPA

What now?

- *To explain the rationale behind the promotion of the CPA approach.*
- *To consider implications for the whole school for this approach.*
- *To look at how resources can be used across the year groups.*

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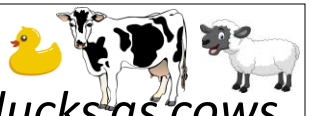
Double sided counters for problem solving e.g. ratio

Peter had 10 marbles.
Julie had 3 times as many.
How many more marbles did Julie have than Peter?
What if Peter had 20, 30 or 5 marbles?

A farmer has 42 animals.
There are twice as many ducks as cows and 3 times as many sheep as cows.
How many sheep, cows and ducks?
What if there were 72 animals?
How many animals would there be if there were 9 cows?



A gardener plants tulip bulbs in a flower bed.
She plants 3 red bulbs for every 4 white bulbs.
She plants 60 red bulbs.
How many white bulbs does she plant?
If there were 100 white bulbs, how many red bulbs would she plant?