Effective Teaching of Multiplication Bonds

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By the end of this week I shall expect every one of you to know the two times tables by heart. And in a year's time I hope you will know all the multiplication tables up to twelve. It will help you enormously if you do!

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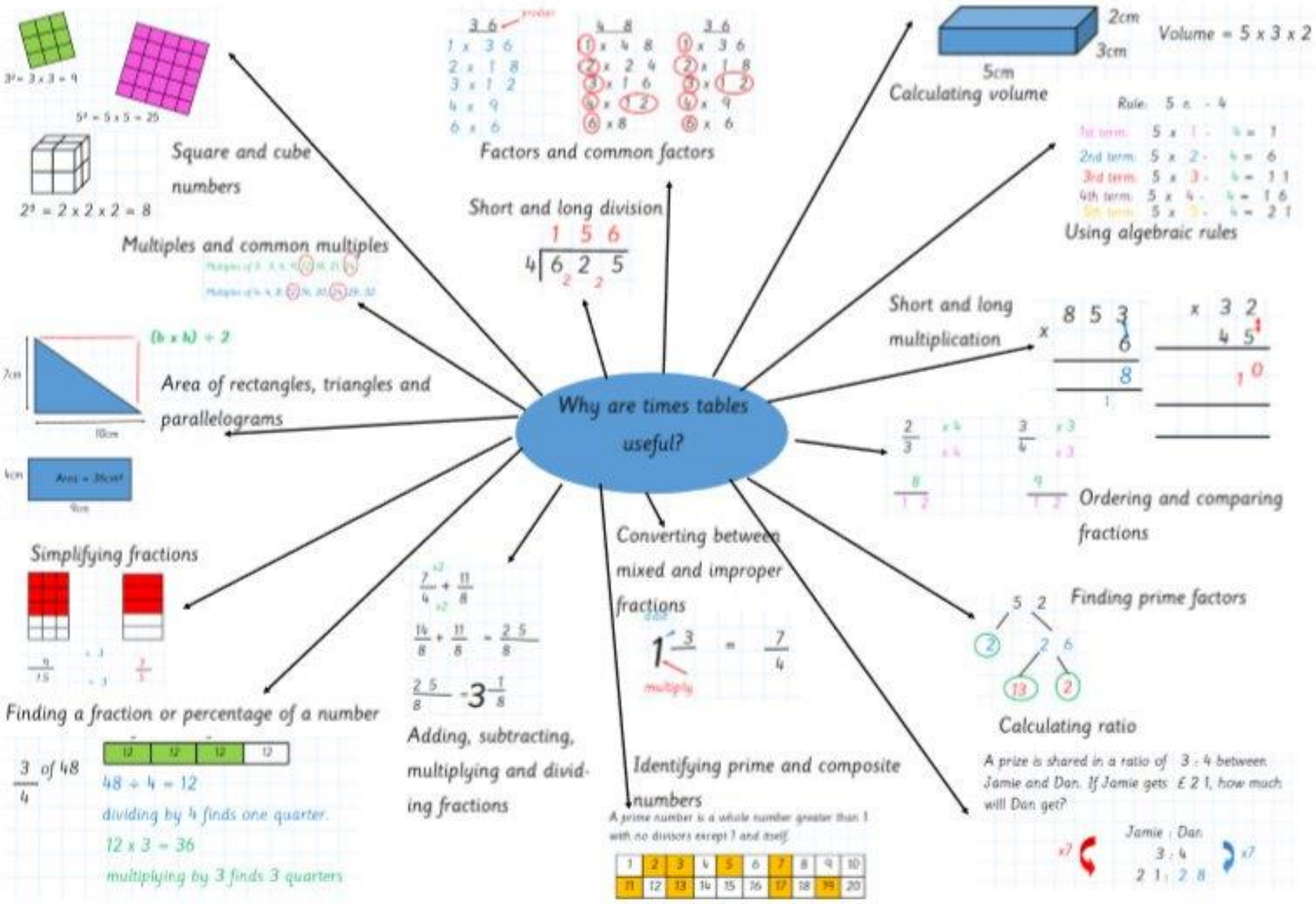

https://twitter.com/SarahFarrellKS2

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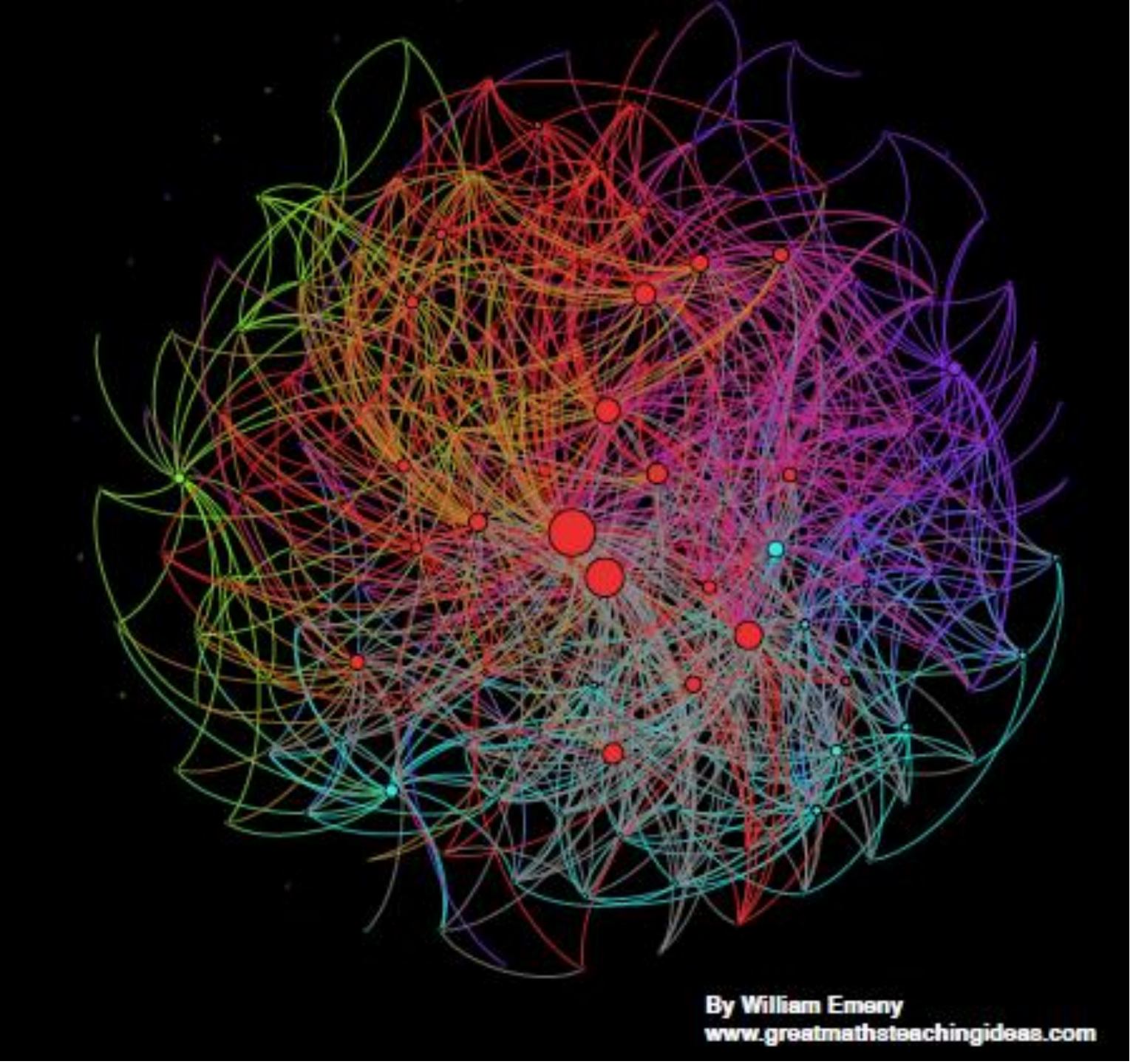

In ranked order, the most important topics for students to master for GCSE, based on the number of topics they are prior knowledge for are as follows:

- Multiply and divide whole numbers (This is prior knowledge for 90 topics)
- Add and subtract whole numbers (73)
- BIDMAS (50)
- Multiply and divide decimal numbers (43)

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2016

21% unanswered

46% incorrect

33% correct

21

$$5{,}542 \div 17 = 326$$

Explain how you can use this fact to find the answer to **18 × 326**

1 mark

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The umm-ers

The counter-oners

The best guessers

The fact barkers

The tables ninjas

Tell me about times tables in your school…

- “The children have Times Table Rock Stars.”
- “We do tables tests on a Friday.”

“Pedagogy trumps curriculum every time. It is very clear that the way you teach and how you teach is always more powerful than just changing the curriculum.”

Dr K. Collins

(CEO Educational Endowment Foundation)

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FACTUALLY FLUENT WITH AUTOMATICITY

CONCEPTUAL UNDERSTANDING

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Number Sense – building conceptual understanding

"Teachers should help students develop maths facts, not by emphasizing facts for the sake of facts or using 'timed tests' but by encouraging students to use, work with and explore numbers. As students work on meaningful number activities they will commit math facts to heart at the same time as understanding numbers and math."

Jo Boaler, Fluency without fear 2015

https://www.youcubed.org/resources/what-is-number-sense/

"Those who learned through strategies achieved 'superior performance' over those who memorized, they solved problems at the same speed, and showed better transfer to new problems." Delazer et al, 2005

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Jenny Field – Whole School Intervention for teaching, learning and understanding times tables

Component 1: Regular retrieval practice to develop fluency (5-10 minutes 3-5 times a week)

Component 2: 2-3 dedicated maths lessons every term to explore each times table

https://www.ncetm.org.uk/features/whole-school-approach-to-learning-times-tables/

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Equal groups concept starts small!

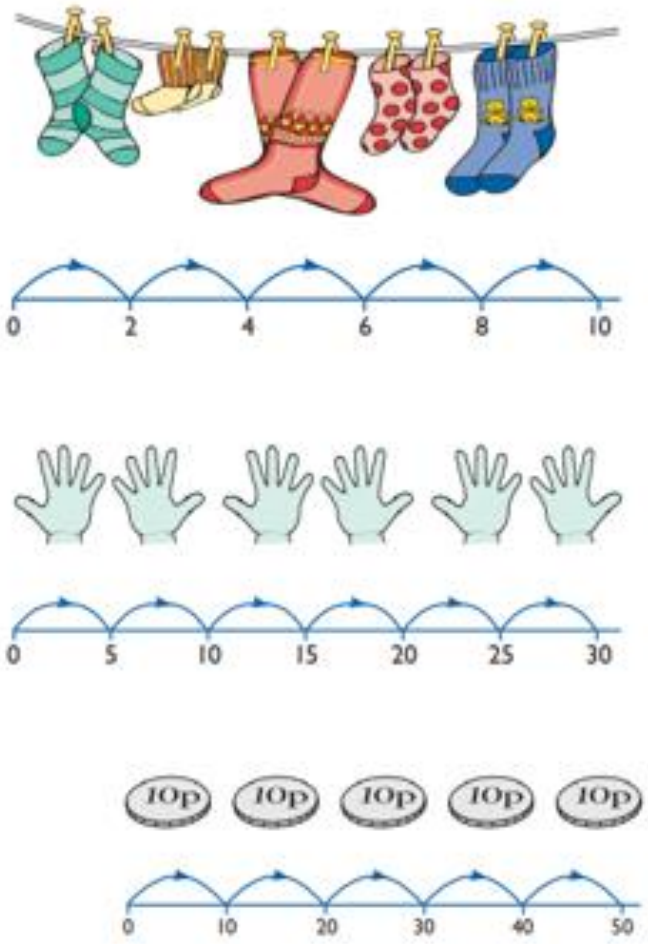
Number ELG:

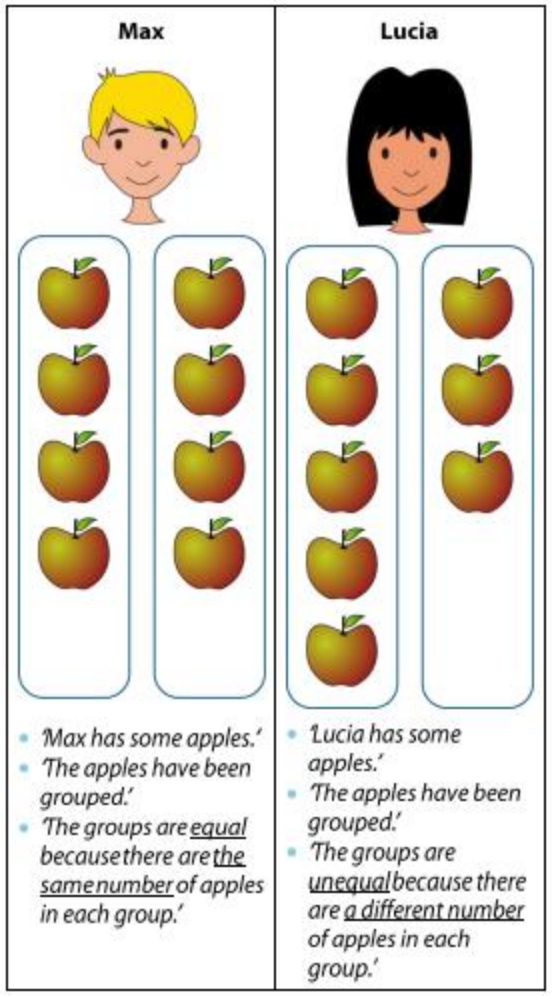
- Have a deep understanding of number to 10, including the composition of each number;
- Subitise (recognise quantities without counting) up to 5;
- Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 (including subtraction facts) and some number bonds to 10, including double facts.

Numerical Patterns

- Verbally count beyond 20, recognising the pattern of the counting system;
- Compare quantities up to 10 in different contexts, recognising when one quantity is greater than, less than or the same as the other quantity;
- Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally.

Key Concept: Equal Groups





- 'How many equal groups are there?'
- 'How many cakes are there in each group?'

- 'There are five equal groups of cakes.'
- 'There are three cakes in each group.'
- 'There are five groups of three.'

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Unitising links to repeated addition

1 box = 10 eggs

1 egg thirty times

or

10 eggs three times

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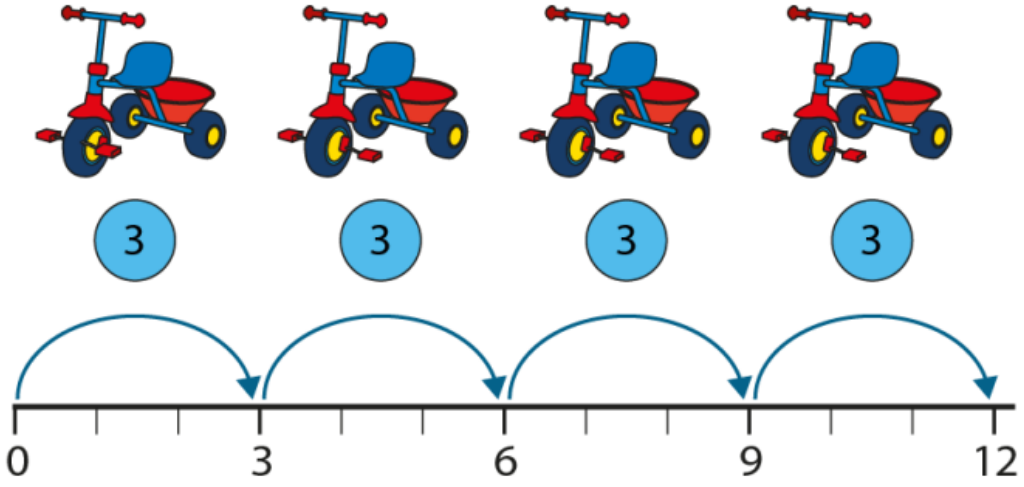
Key resource: Models and Representations

'How many wheels are there? Count in groups of three.'



- *'Three, six, nine, twelve. There are twelve wheels.'*
- *'There are four groups of three; there are twelve altogether.'*
- *'There are three, four times; there are twelve altogether.'*

$4 \times 3 = 12$ $\qquad$ $3 \times 4 = 12$

- *'Four is a factor.'*
- *'Three is a factor.'*
- *'The product of four and three is twelve'*
- *'Twelve is the product of four and three.'*

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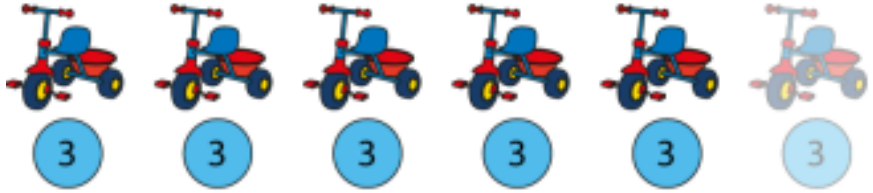
Number line:

Building up the three times table:



$0 \times 3 = 0$	$3 \times 0 = 0$
$1 \times 3 = 3$	$3 \times 1 = 3$
$2 \times 3 = 6$	$3 \times 2 = 6$
$3 \times 3 = 9$	$3 \times 3 = 9$
$4 \times 3 = 12$	$3 \times 4 = 12$
$5 \times 3 = 15$	$3 \times 5 = 15$
$6 \times 3 = 18$	**$3 \times 6 = 18$**

	× 3
0	0
1	3
2	6
3	
4	12
5	15
6	18
7	
8	24
9	27
10	30
11	33
12	36

↓ + 3

↑ − 3

- *'If there are nine tricycles, how many wheels are there altogether?'*
- *'How many tricycles are there if there are twenty-one wheels?'*
- *'If the product is thirty, what are the factors?'*
- *'Why are eight times three and three times eight both equal to twenty-four?'*

Key Concept: Rote and Skip Counting

$1 \times 7 = 7$
$2 \times 7 = 14$
$3 \times 7 = 21$
$4 \times 7 = 28$
$5 \times 7 = 35$
$6 \times 7 = 42$
$7 \times 7 = 49$
$8 \times 7 = 56$
$9 \times 7 = 63$
$10 \times 7 = 70$
$11 \times 7 = 77$
$12 \times 7 = 84$

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Interpreting Questions

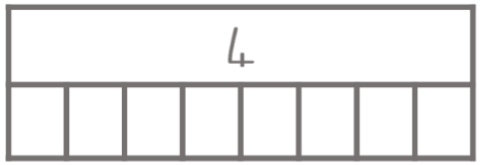
Which picture?

Draw lines to match the questions to the bar models:

4 friends share 8 cherries.
How many cherries each?

4 pizzas shared by 8 friends.
How much pizza each?

4 friends each have 8 sweets.
How many in total?



Adam buys **6** bags of white balloons.

Chen buys **3** bags of red balloons.

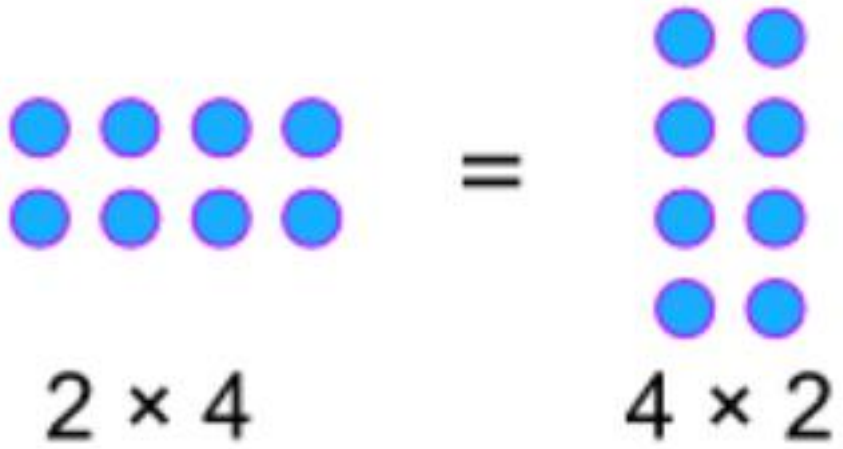
Adam says,

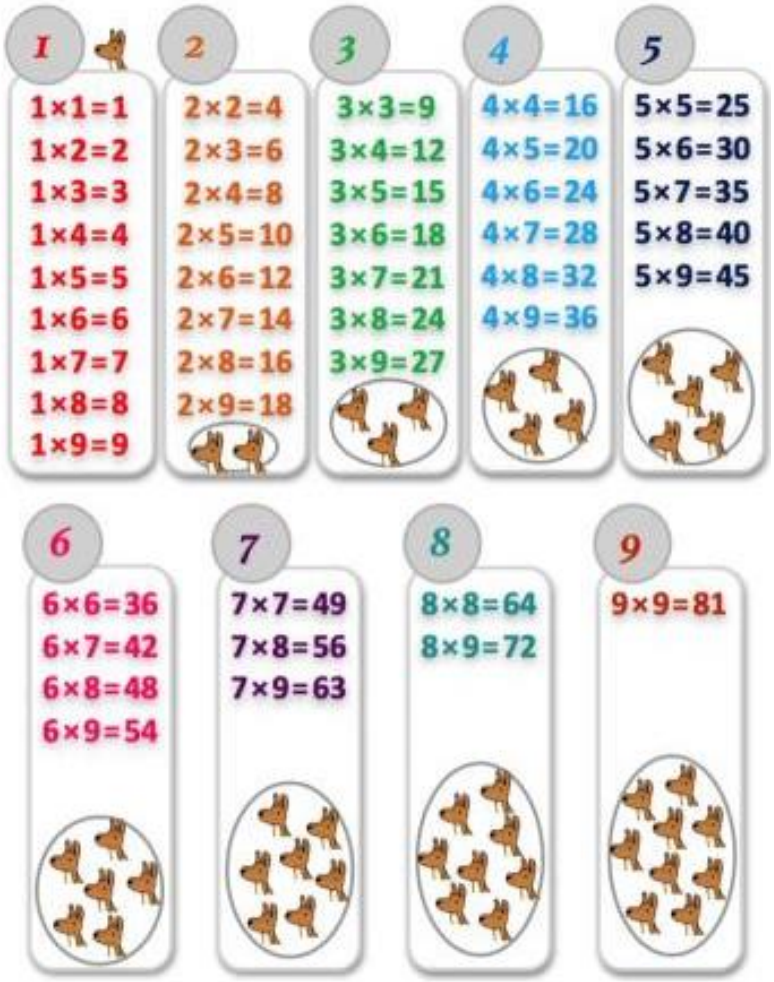
'I have four times as many balloons as Chen.'

Explain why Adam is correct.

Commutative Law

When you swap numbers over, you still get the same answer in addition and multiplication.





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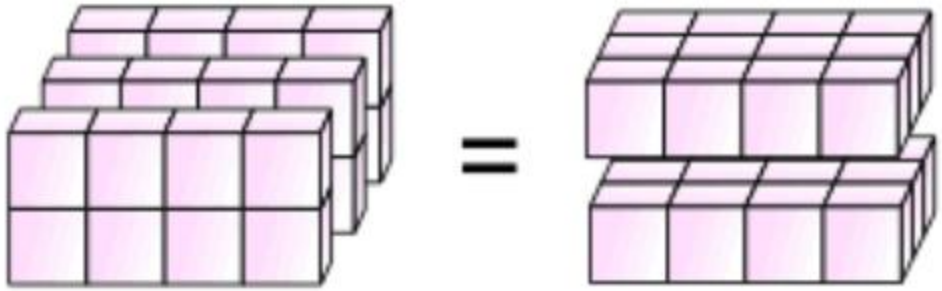
Associative Law

It doesn’t matter how we group the numbers in addition or multiplication.



$(2 \times 4) \times 3$ $\qquad$ $2 \times (4 \times 3)$

$2 \times 16 \times 5 = (2 \text{ x } 5) \text{ x } 16$

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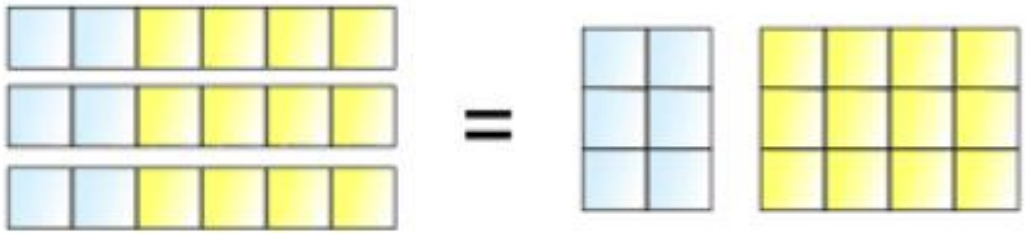
Distributive Law

Can be thought of as a type of partitioning



3 x (2+4) = 3x2 + 3x4

3 lots of **(2+4)** is the same as **3 lots of 2** plus **3 lots of 4**

$6 \times 204 =$ (6 x 200) + (6 x 4)

$26 \times 3 - 24 \times 3 =$ (26 – 24) x 3

$6\times7 + 2\times7 + 3\times7 + 5\times7 + 4\times7 =$ (6+2+3+5+4) x7

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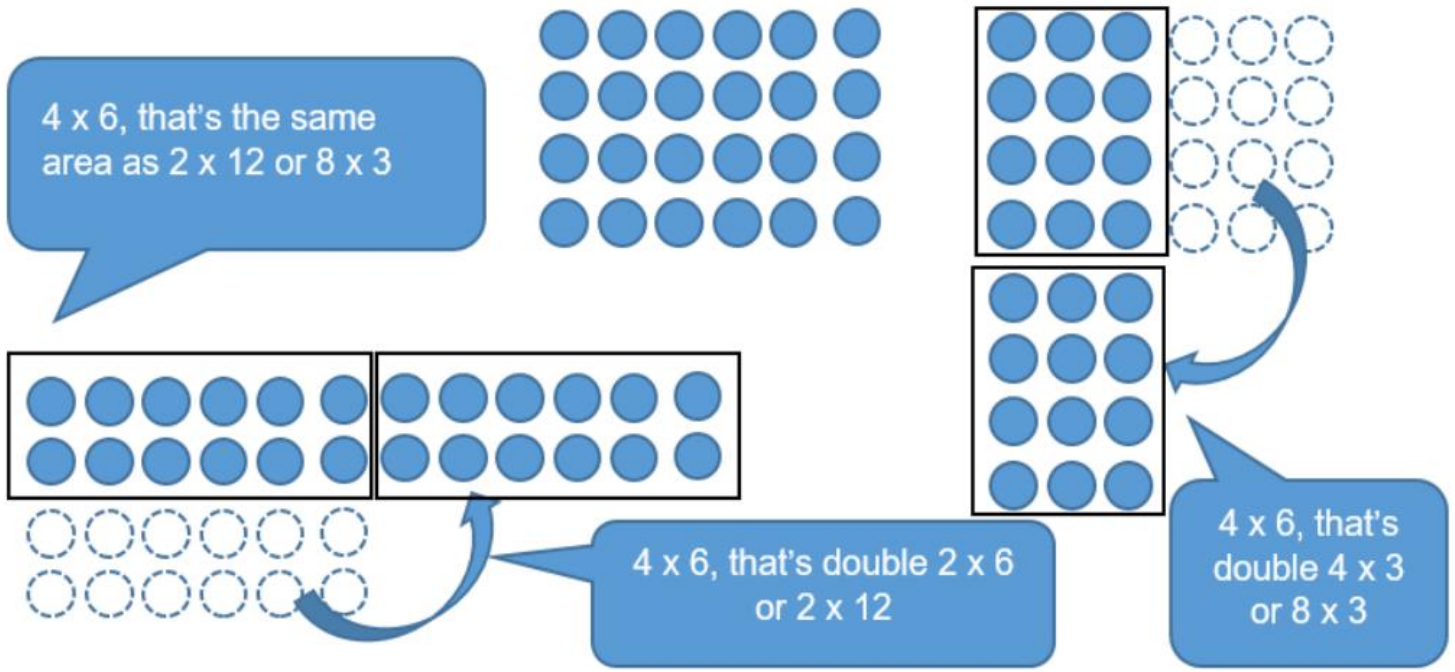
Conservation of Area

Calculations could be made easier by re-arranging our array.



https://www.ncetm.org.uk/teaching-for-mastery/mastery-materials/primary-mastery-professional-development/multiplication-and-division/

Spine 2 Overview

Teacher guides | Years 1–6

Year 1

2.1 Counting, unitising and coins

Year 2

2.2 Structures: multiplication representing equal groups
2.3 Times tables: groups of 2 and commutativity (part 1)
2.4 Times tables: groups of 10 and of 5, and factors of 0 and 1
2.5 Commutativity (part 2), doubling and halving
2.6 Structures: quotitive and partitive division

Year 3

2.7 Times tables: 2, 4 and 8, and the relationship between them
2.8 Times tables: 3, 6 and 9, and the relationship between them
2.9 Times tables: 7 and patterns within/across times tables

Year 4

2.10 Connecting multiplication and division, and the distributive law
2.11 Times tables: 11 and 12
2.12 Division with remainders
2.13 Calculation: multiplying and dividing by 10 or 100
2.14 Multiplication: partitioning leading to short multiplication
2.15 Division: partitioning leading to short division

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2.8 Times tables: 3, 6 and 9, and the relationship between them

Build up the three/six/nine times table; using different structures/interpretations of multiplication and division, solve problems related to these tables; explore connections between the three, six and nine times tables.

- **Teaching point 1:** Counting in multiples of three can be represented by the three times table. Adjacent multiples of three have a difference of three. Facts from the three times table can be used to solve multiplication and division problems with different structures.
- **Teaching point 2:** Counting in multiples of six can be represented by the six times table. Adjacent multiples of six have a difference of six. Facts from the six times table can be used to solve multiplication and division problems with different structures.
- **Teaching point 3:** Products in the six times table are double the products in the three times table; products in the three times table are half of the products in the six times table.
- **Teaching point 4:** Counting in multiples of nine can be represented by the nine times table. Adjacent multiples of nine have a difference of nine. Facts from the nine times table can be used to solve multiplication and division problems with different structures.
- **Teaching point 5:** Products in the nine times table are triple the products in the three times table. Products that are in the three, six and nine times tables share the same factors.
- **Teaching point 6:** Divisibility rules can be used to find out whether a given number is divisible (to give a whole number) by three, six or nine.

JENNY FIELD'S SUGGESTION

Year	AUT 1	AUT 2	SPR 1	SPR 2	SUM 1	SUM 2
1	Experience of counting in 2s, 5s and 10s					
2	x1	(x1) x2	x5	(x5) x10	x0	revision
3	(x2) x4	(x4) x8	x3	(x3) x6	(x6) x12	revision
4	x9	x7	x11	squares	revision	Revision

Why focus on one TT per half term? Plasticity of the brain

Neuroscientists tell us it takes approximately 8 weeks of repetition to make a new neural pathway with a myelin sheath – making this the 'go to' automated thinking. Then continued practised makes the sheath thicker which makes it stronger.

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Building Maths Muscles

"The real secret to learning times tables and getting them to 'stick' comes through hard work but work filled with special ingredients: jokes, giggles, movements, puzzles, music, games, team work and novelty. If we can fill times tables with fun, curiosity, surprises and memories then we are giving children every chance of growing maths muscles who can answer 7x8 without dissolving into tears."

John Dabell, Rising Stars

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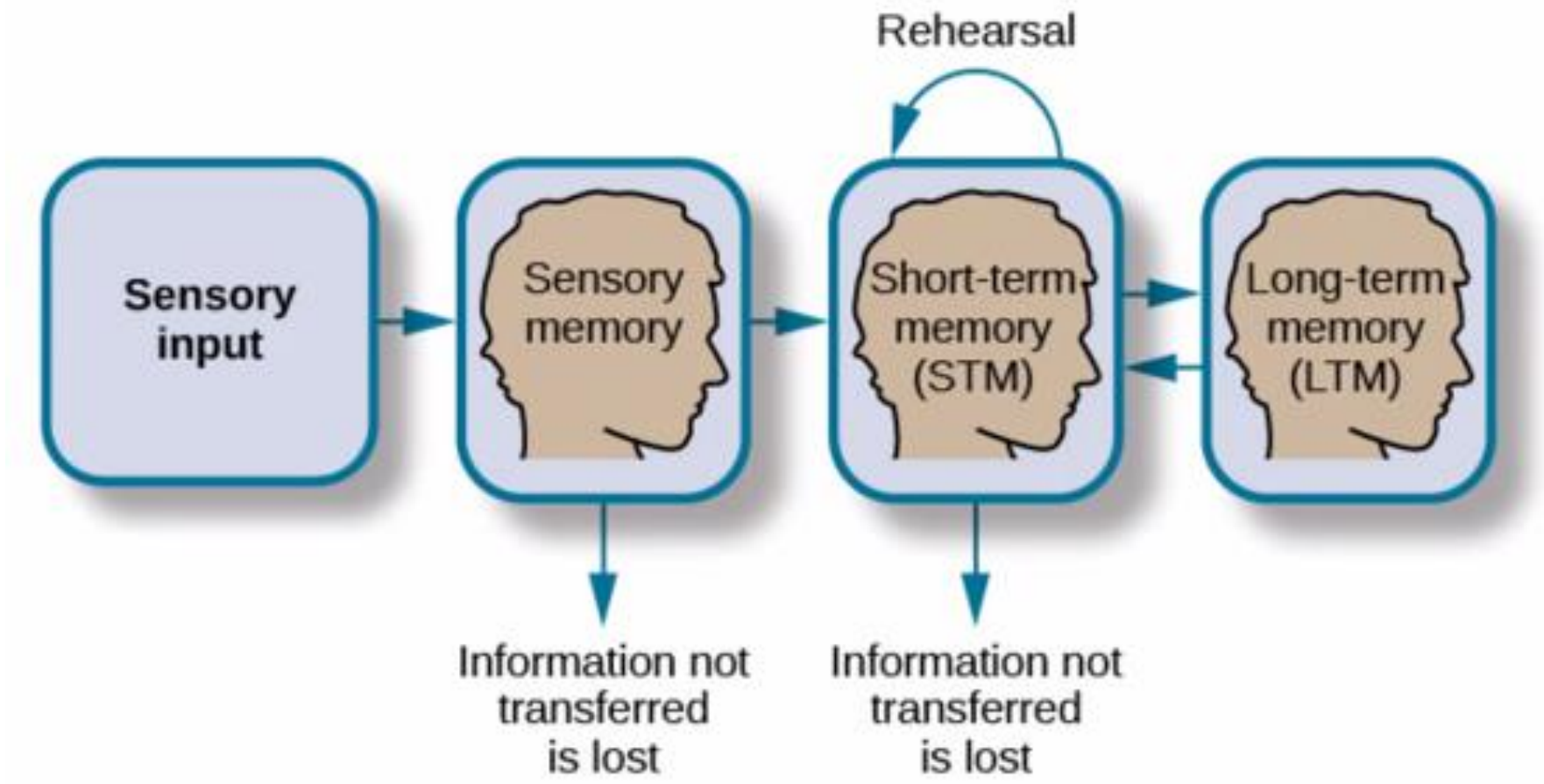
We learn what we think about



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Memorisation vs automaticity

"Memorisation of basic facts usually refers to committing the result of operations to memory so that thinking is unnecessary.

Teaching facts for automaticity in contrast **relies on thinking**. Answers to facts must be automatic but thinking about the relationships among the facts is critical. A child can then think of 9x6 as (10x6) – 6."

Twomey Fosnot, C and Dolk, M (2000 pg85)

2019 PISA research demonstrates that the UK has the highest use of **memorisation, rehearsal and repetition**; using 'learning by heart' as a strategy. However, the UK is lowest in lessons involving deep learning and critical thinking and solving non-routine problems.

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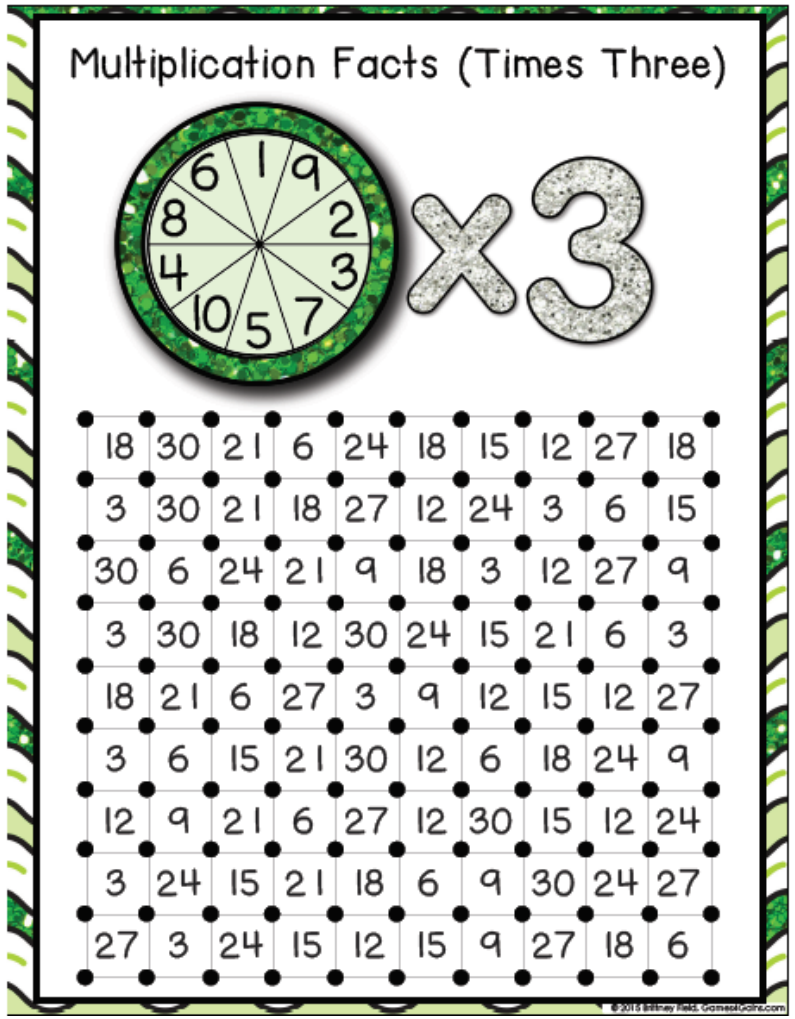

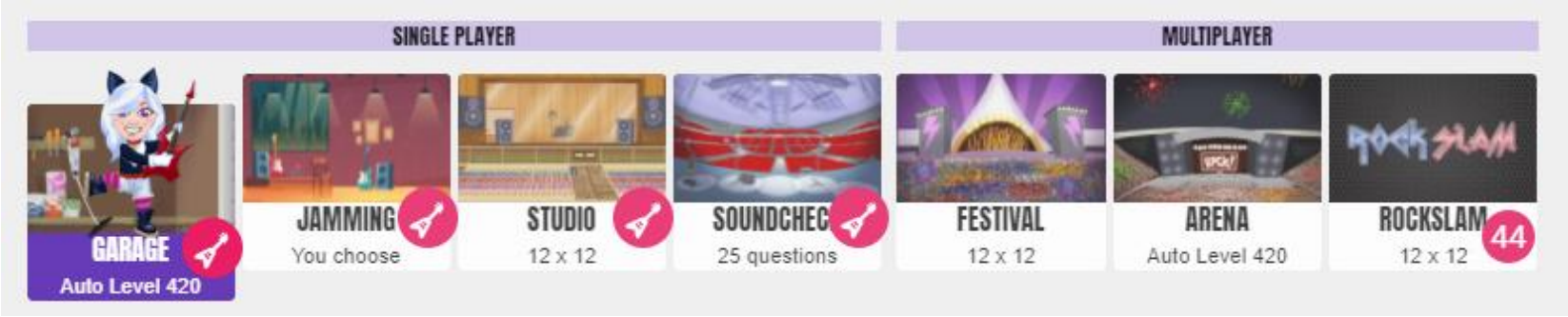

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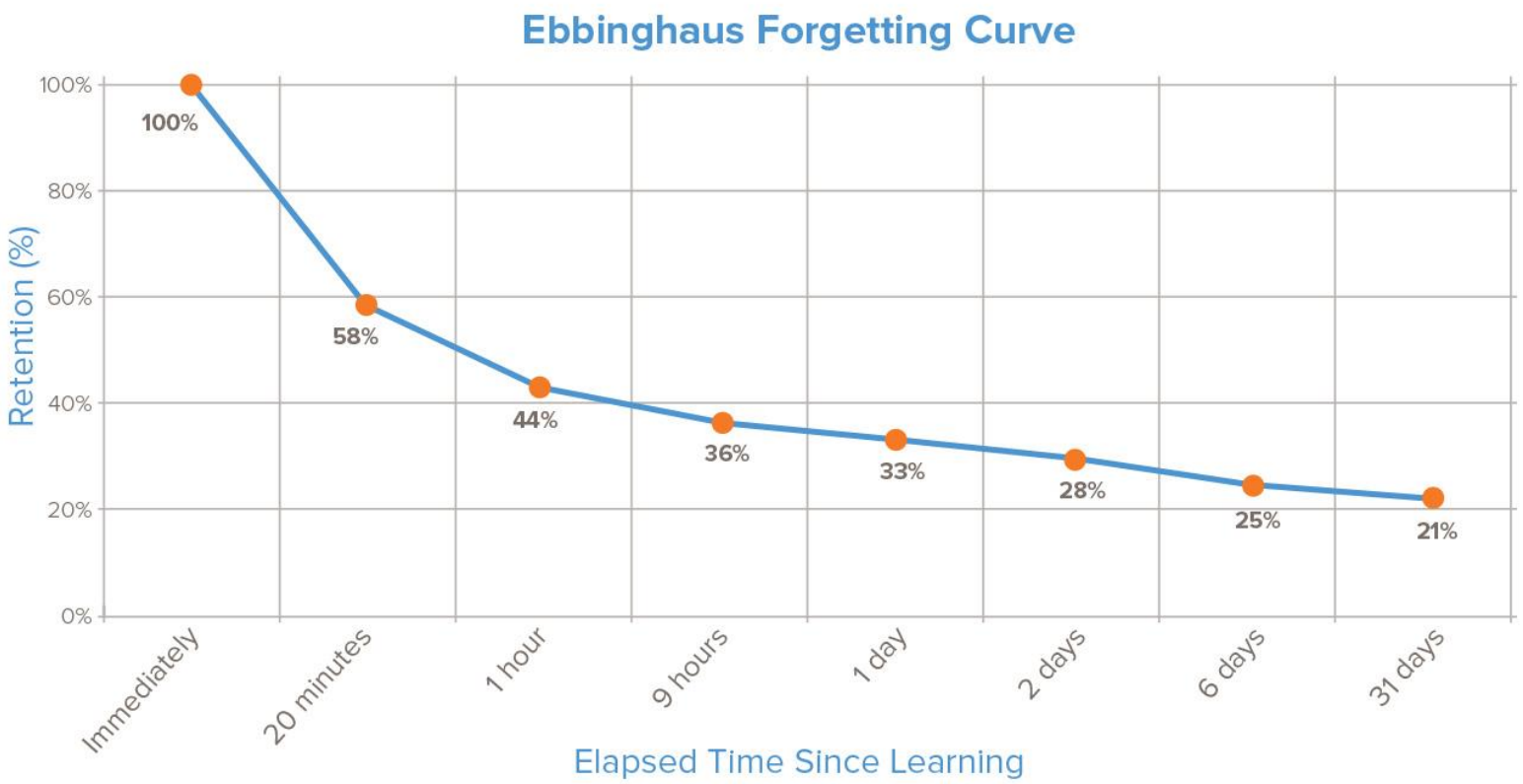
Spacing and Interleaving

- The more you retrieve a memory, the more memorable it becomes.
- Small, routine, low stakes tasks, regularly done. ‘Learning feedback’ as opposed to assessments/tests
- Allow them to almost forget before seeing what stuck!



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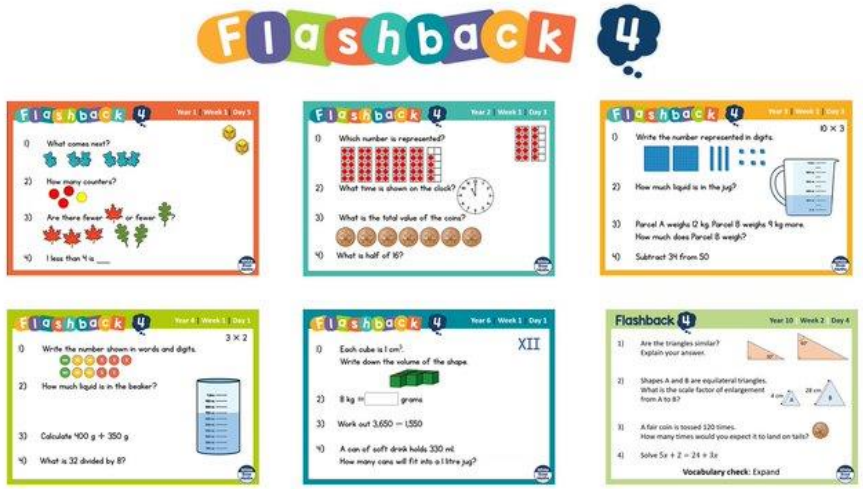
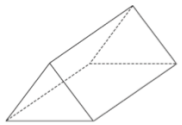
Practise and Revision



A question from last session	A question from last week	A question from last unit

Name: _____ Primary 5-a-day Bronze

3rd January

$55 \div 5$	20×30
60 —is 15 more than→ 45 ☐ —is 15 more than→ 77	
How many **edges** does a triangular prism have?	
Mr Smith had 95 Christmas cards to post. He has posted 48 so far. How many Christmas cards does he have left to post?	

 www.corbettmaths.com

1		2	
3 x 10 = ____	6 x 10 = ____	5 x 10 = ____	10 x 10 = ____
30 ÷ 10 = ____	50 ÷ 10 = ____	3 x 10 = ____	10 x 2 = ____
12 x 10 = ____	9 x 10 = ____	10 x 12 = ____	60 ÷ 10 = ____
10 x 3 = ____	10 x 10 = ____	100 ÷ 10 = ____	4 x 10 = ____
10 x 4 = ____	40 ÷ 10 = ____	7 x 10 = ____	10 x 5 = ____
10 x 11 = ____	10 x 12 = ____	9 x 10 = ____	12 x 10 = ____
11 x 10 = ____	10 x 2 = ____	10 x 9 = ____	50 ÷ 10 = ____
3 x 10 = ____	2 x 10 = ____	10 x 11 = ____	10 x 10 = ____
80 ÷ 10 = ____	100 ÷ 10 = ____	10 x 2 = ____	10 x 4 = ____
10 x 2 = ____	10 x 9 = ____	30 ÷ 10 = ____	110 ÷ 10 = ____
10 x 4 = ____	5 x 10 = ____	10 x 10 = ____	10 x 7 = ____
10 x 5 = ____	70 ÷ 10 = ____	80 ÷ 10 = ____	10 x 8 = ____
60 ÷ 10 = ____	8 x 10 = ____	2 x 10 = ____	9 x 10 = ____
10 x 7 = ____	10 x 12 = ____	3 x 10 = ____	10 x 6 = ____
6 x 10 = ____	10 x 10 = ____	7 x 10 = ____	80 ÷ 10 = ____
10 x 7 = ____	110 ÷ 10 = ____	6 x 10 = ____	11 x 10 = ____
9 x 10 = ____	7 x 10 = ____	40 ÷ 10 = ____	90 ÷ 10 = ____
80 ÷ 10 = ____	12 x 10 = ____	10 x 3 = ____	2 x 10 = ____
10 x 10 = ____	20 ÷ 10 = ____	8 x 10 = ____	10 x 11 = ____
4 x 10 = ____	10 x 5 = ____	10 x 4 = ____	10 x 5 = ____

All 10 times table facts

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Jenny's Eight Recommendations…

1) Decide on the order for teaching and for embedding

2) Clear rationale for language: 6 x 7 (MTC says in the x6) – important in problem although not in abstract. Common vocab across school.

3) Systematically build new tables around facts they already know

4) Introduce new table through conceptual link e.g. what comes in 5s?

5) Retrieval practice – bank of activities, conceptual support, verbal patterns in different ways, order and non-order, tests but NOT MAIN ACTIVITY

6) CPA for all children – best representation for the job

7) Explore patterns within table, divisibility and how it relates to others

8) Opportunities to develop mastery through variation (not variety) and intelligent practice.

Thank you

- https://www.ncetm.org.uk/features/whole-school-approach-to-learning-times-tables/

- https://gala.gre.ac.uk/id/eprint/26932/6/26932%20FIELD_A_Whole_School_Intervention_for_Teaching_Learning_and%20Understanding_Jan%202020.pdf

- https://gala.gre.ac.uk/id/eprint/31051/6/31051%20FIELD_Teaching%2C%20Learning%20and%20Understanding_2021.pdf

- https://www.atm.org.uk/write/MediaUploads/News/The_teaching_and_learning_of_multiplication_bonds_ATM_MA_may__22_final.pdf

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