Disciplinary Literacy in Maths

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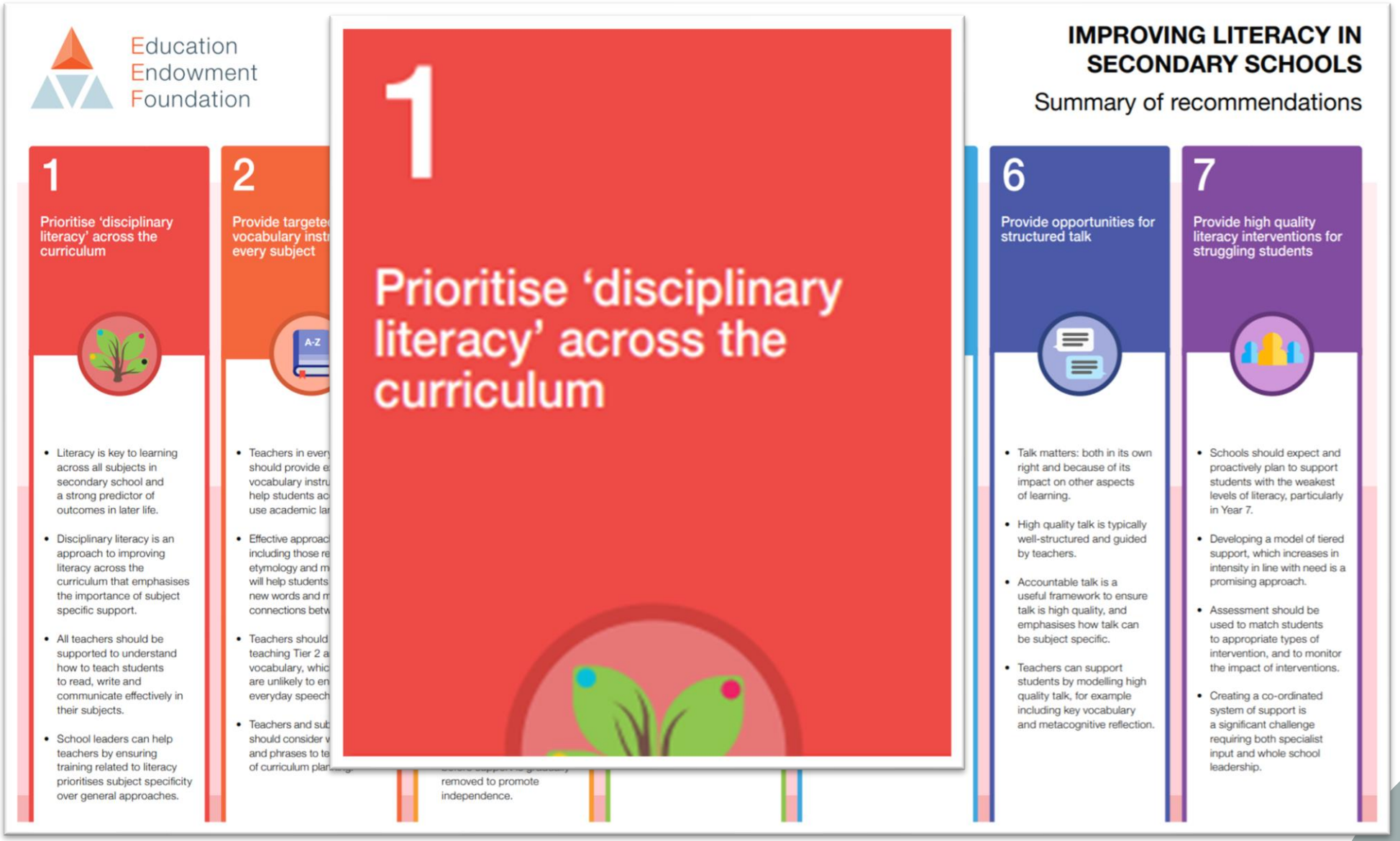
DMAT
MATHSHUBS
SUSSEX

Welcome

- What is disciplinary literacy?
- How does it relate to maths?
- Why is **collaboration** vital?

- Explore four specific areas in more detail:
  - What is our ideal?
  - What are the difficulties?
  - What can help?
  - Takeaways

WHY???


Education
Endowment
Foundation
IMPROVING LITERACY IN SECONDARY SCHOOLS
Summary of recommendations
1
Prioritise 'disciplinary literacy' across the curriculum
Literacy is key to learning across all subjects in secondary school and a strong predictor of outcomes in later life.
Disciplinary literacy is an approach to improving literacy across the curriculum that emphasises the importance of subject specific support.
All teachers should be supported to understand how to teach students to read, write and communicate effectively in their subjects.
School leaders can help teachers by ensuring training related to literacy prioritises subject specificity over general approaches.
2
Provide targete
vocabulary inst
every subject
A-Z
Teachers in ever
should provide e
vocabulary instru
help students ac
use academic la
Effective approac
including those re
etymology and m
will help students
new words and m
connections betw
Teachers should
teaching Tier 2 a
vocabulary, whic
are unlikely to en
everyday speech
Teachers and sub
should consider v
and phrases to te
of curriculum plan
removed to promote independence.
1
Prioritise 'disciplinary literacy' across the curriculum
6
Provide opportunities for structured talk
Talk matters: both in its own right and because of its impact on other aspects of learning.
High quality talk is typically well-structured and guided by teachers.
Accountable talk is a useful framework to ensure talk is high quality, and emphasises how talk can be subject specific.
Teachers can support students by modelling high quality talk, for example including key vocabulary and metacognitive reflection.
7
Provide high quality literacy interventions for struggling students
Schools should expect and proactively plan to support students with the weakest levels of literacy, particularly in Year 7.
Developing a model of tiered support, which increases in intensity in line with need is a promising approach.
Assessment should be used to match students to appropriate types of intervention, and to monitor the impact of interventions.
Creating a co-ordinated system of support is a significant challenge requiring both specialist input and whole school leadership.

What is disciplinary literacy?

"…an approach to improving literacy across the curriculum. It recognises that literacy skills are both general and subject specific, emphasising value of supporting teachers of every subject to teach students how to read, write and communicate effectively"

"each subject has its own unique language, ways of knowing, doing and communicating"

EEF Guidance Report

What is disciplinary literacy?

Knowledge of content + Knowledge of how to think about and communicate that content = Subject knowledge

Disciplinary literacy in maths

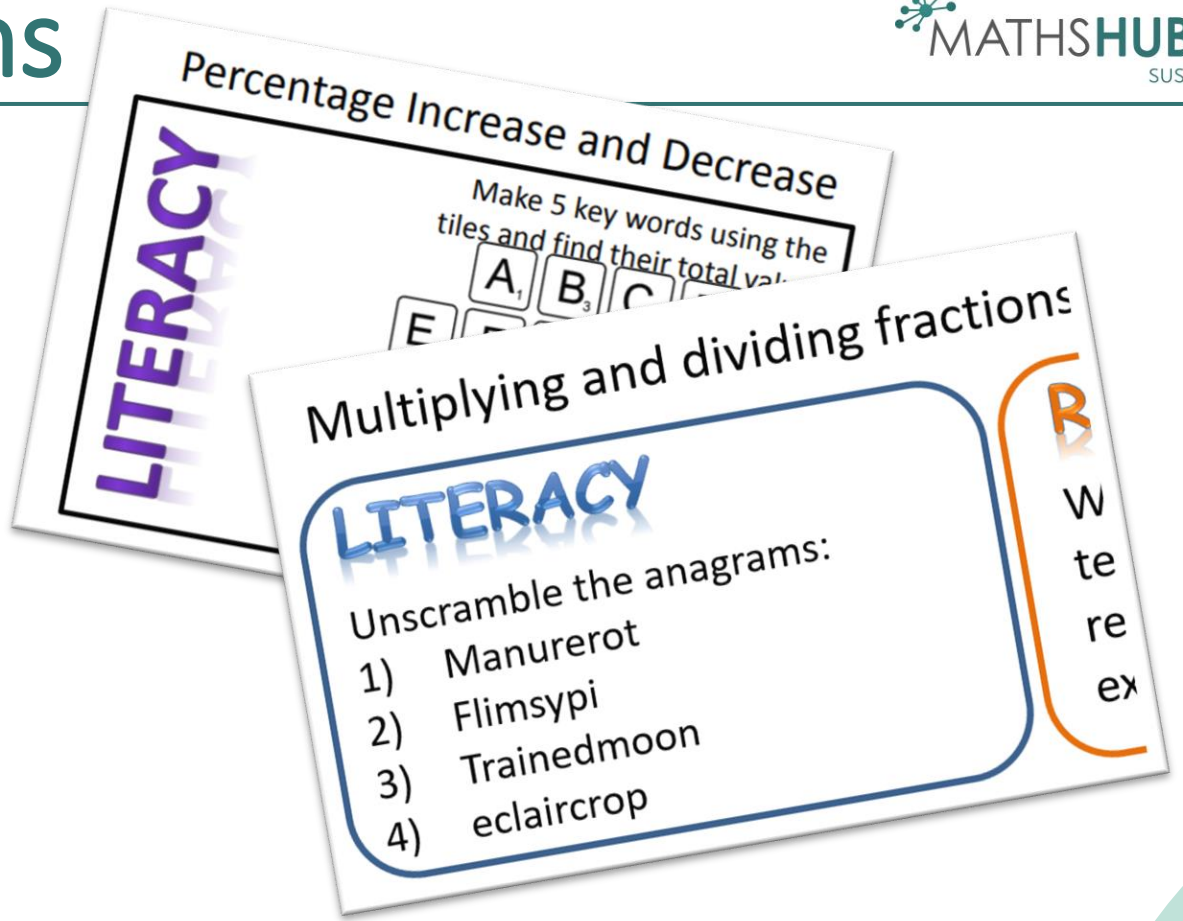
Not literacy **within** maths, but literacy **of** maths

"…disciplinary learning doesn't just build knowledge but actually produces and constructs it"

"Disciplinary Literacy: A Shift that Makes Sense" ReLeah Lent 2017

Disciplinary literacy in maths

- Keywords
  eg @MissBsResources

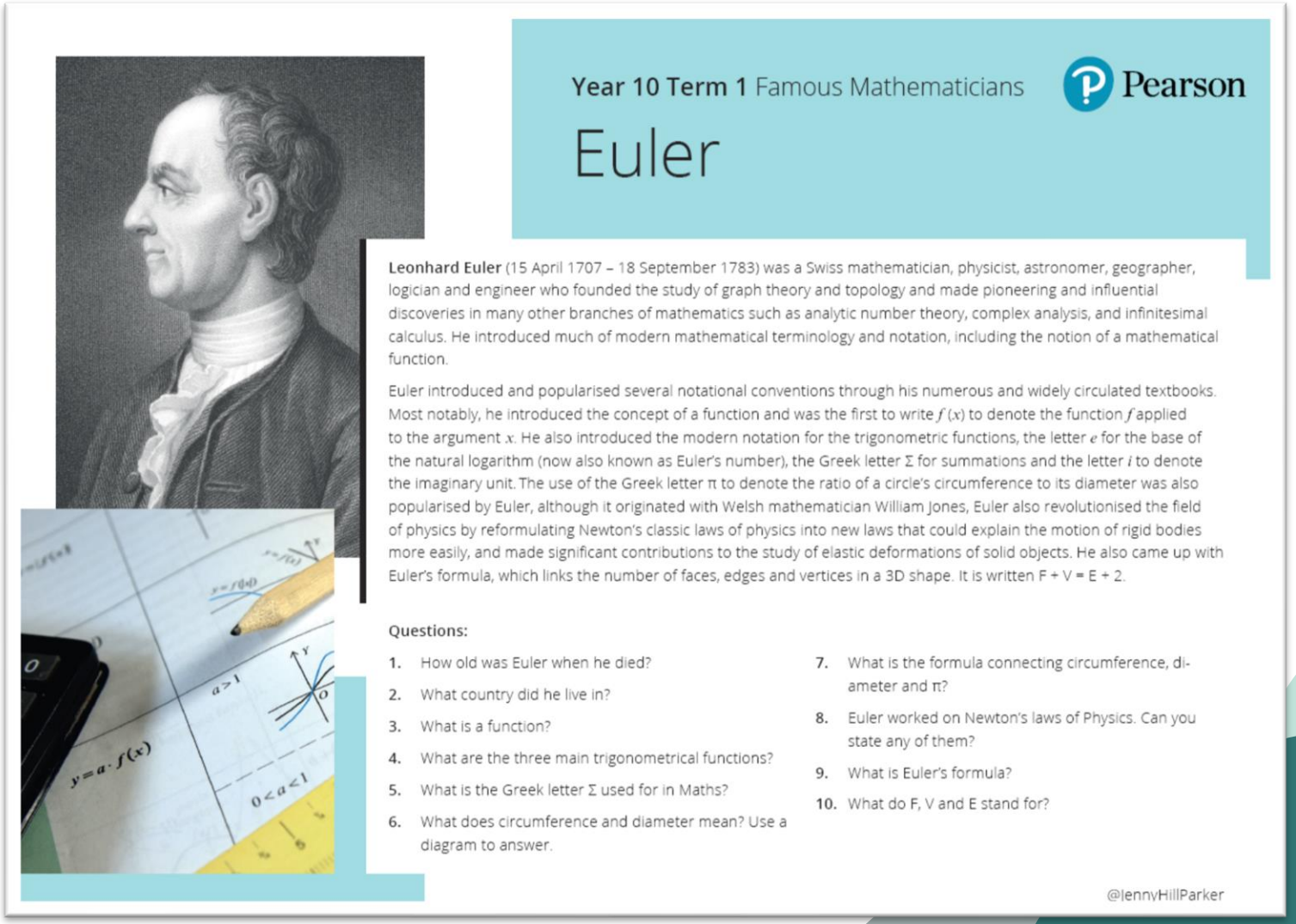


Disciplinary literacy in maths

- Keywords
  eg @MissBsResources

- Comprehension tasks
  eg @JennyHillParker and Pearson

BUT…
could we do more?



Year 10 Term 1 Famous Mathematicians

Pearson

Euler

Leonhard Euler (15 April 1707 – 18 September 1783) was a Swiss mathematician, physicist, astronomer, geographer, logician and engineer who founded the study of graph theory and topology and made pioneering and influential discoveries in many other branches of mathematics such as analytic number theory, complex analysis, and infinitesimal calculus. He introduced much of modern mathematical terminology and notation, including the notion of a mathematical function.

Euler introduced and popularised several notational conventions through his numerous and widely circulated textbooks. Most notably, he introduced the concept of a function and was the first to write $f(x)$ to denote the function f applied to the argument x. He also introduced the modern notation for the trigonometric functions, the letter e for the base of the natural logarithm (now also known as Euler's number), the Greek letter Σ for summations and the letter i to denote the imaginary unit. The use of the Greek letter π to denote the ratio of a circle's circumference to its diameter was also popularised by Euler, although it originated with Welsh mathematician William Jones, Euler also revolutionised the field of physics by reformulating Newton's classic laws of physics into new laws that could explain the motion of rigid bodies more easily, and made significant contributions to the study of elastic deformations of solid objects. He also came up with Euler's formula, which links the number of faces, edges and vertices in a 3D shape. It is written F + V = E + 2.

Questions:

1. How old was Euler when he died?
2. What country did he live in?
3. What is a function?
4. What are the three main trigonometrical functions?
5. What is the Greek letter Σ used for in Maths?
6. What does circumference and diameter mean? Use a diagram to answer.
7. What is the formula connecting circumference, diameter and π?
8. Euler worked on Newton's laws of Physics. Can you state any of them?
9. What is Euler's formula?
10. What do F, V and E stand for?

@JennyHillParker

EEF Guidance Report

Misconceptions

Literacy across the curriculum ≠ Successful application of literacy in a subject discipline

Misconceptions

- Not reading ABOUT your subject, but reading IN your subject

"a **text** is really anything imbued with meaning"

Draper 2015

Lent (2016)

What are Literacies within the Disciplines? The following lists for each of the major content areas, while not comprehensive, can act as starting points through which communities of teachers can begin to think in terms of disciplinary literacy (Lent, 2016).

	Read	Write	Think
Science	*When scientists read, they* • Ask "Why?" more than "What?" • Interpret data, charts, illustrations • Seek to understand concepts and words • Determine validity of sources and quality of	*When scientists write, they* • Use precise vocabulary • Compose in phrases, bullets, graphs, or sketches • Use passive voice	*When scientists think, they* • Tap into curiosity to create questions • Rely on prior knowledge or research • Consider new hypotheses or evidence • Propose explanations
Math	*When mathematicians read, they* • Use information to piece together a solution • Look for patterns and relationships • Decipher symbols and abstract ideas • Ask questions • Apply mathematical reasoning	*When Mathematicians write, they* • Explain, justify, describe, estimate or analyze • Favor calculations over words • Use precise vocabulary • Include reasons and examples • Utilize real-word situations	*When Mathematicians think, they* • Consider patterns • Utilize previous understandings • Find connections • Estimate, generalize, and find exceptions • Employ mathematical principles
English Language Arts	*When students of English read, they* • Understand how figurative language works • Find underlying messages that evolve as theme • Assume a skeptical stance • Pay attention to new vocabulary or words used in new ways • Summarize and synthesize	*When students of English write, they* • Engage in a process that includes drafting, revising, and editing • Use mentor texts to aid their writing craft • Pay attention to organization, details, elaboration and voice • Rely on the feedback of others • Avoid formulaic writing	*When students of English think, they* • Reflect on multiple texts • Ask questions of the author • Consider research or others ideas • Discuss ideas and themes • Argue both sides of a point

Task to do with your department:

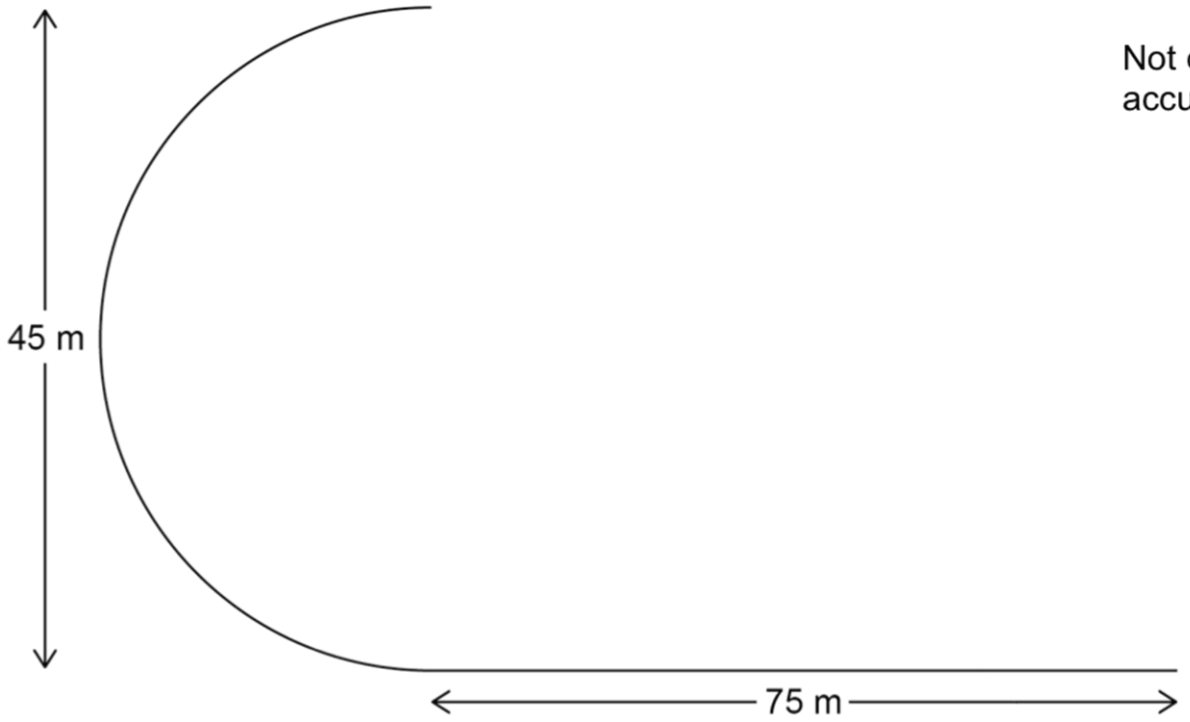
- Take an exam question

- Now consider:
  - What was most important, finding the answer or presenting the solution?
  - Did you use any drawings / representations?
  - How did you decide to set out your working?
  - Did you annotate?
  - Would you show your workings to students as "ideal"?

Part of a running track is the arc of a semicircle joined to a straight line.

The semicircle has diameter 45 metres.

The straight line has length 75 metres.



Not drawn accurately

Abby runs once along this part of the track in 18 seconds.

Work out her average speed.

Give your answer to 2 significant figures.

[4 marks]

The vertices of a re

radius 5 cm

Not drawn accurately

Work out the shade

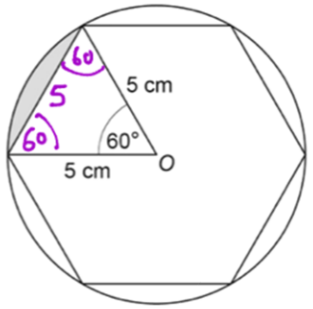
Give your answer ir

integers.

[4 marks]

The vertices of a regular hexagon lie on a circle with centre O and radius 5 cm

Not drawn accurately



Work out the shaded area.

Give your answer in the form $\frac{a\pi - b\sqrt{c}}{12}$ where a, b and c are integers.

[4 marks]

$r = 5$ Area of circle $= \pi r^2$

$= 25\pi$

Area of 60° sector $= \frac{1}{6} \times 25\pi = \frac{25}{6}\pi$

Area of $\triangle = \frac{1}{2}ab\sin C$

$= \frac{1}{2} \times 5 \times 5 \sin 60$

$= \frac{1}{2} \times 25 \times \frac{\sqrt{3}}{2} = \frac{25\sqrt{3}}{4}$

So shaded area $= \frac{25\pi}{6} - \frac{25\sqrt{3}}{4}$

$= \frac{50\pi - 75\sqrt{3}}{12}$

Answer $\frac{50\pi - 75\sqrt{3}}{12}$ cm^2

Four areas of interest

DISCIPLINARY LITERACY IN MATHS

vocab and language

writing mathematically

comprehension and understanding

talking mathematically

Vocabulary and language

Vocabulary and language

Vocabulary and language

What does our ideal student do?

- Understands (and uses) maths-specific vocabulary
- Uses the origin and structure of words to help make connections
- Realises that mathematical language is very precise

What are the difficulties?

- Definitions sometimes unclear

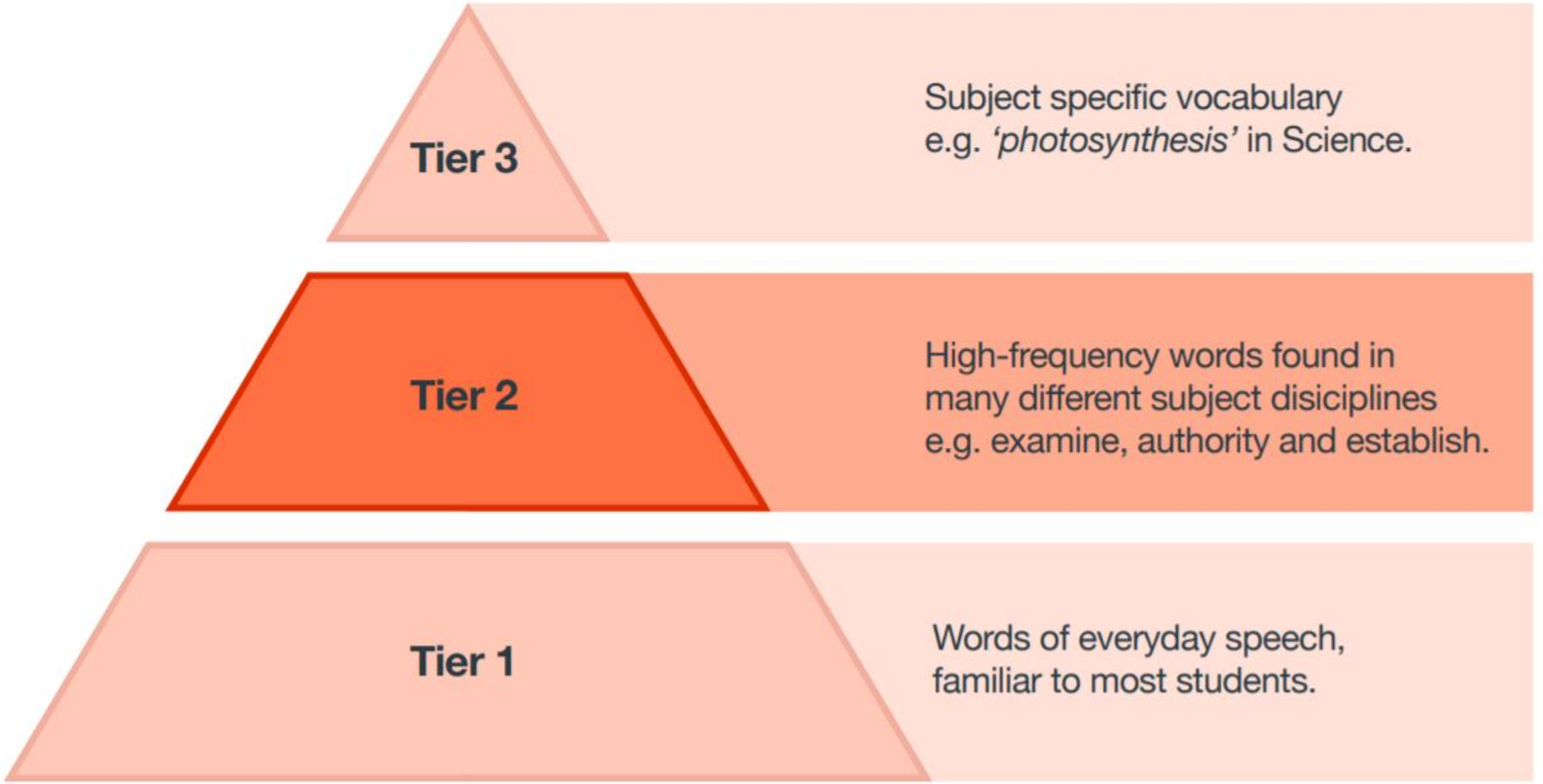
variable *unknown* *factor*

Skills

(www.kerboodle.com, Activate 1, pg. 2/3)

A **variable** is something that can change. It is sometimes called a **factor**. In an investigation you select values for one variable and measure what happens to another. The variable that you change is called the **independent variable**. The variable that you **measure** is called the **dependent variable**. The dependent variable *depends* on the independent one.

Tier 1, 2, 3 vocabulary



Maths specific vocabulary

1. Words which have the same or roughly the same meaning in both contexts (e.g. fewer, between)
2. Words which occur in mathematics and ordinary English, but involve different meanings in these two contexts (e.g. difference, volume)
3. Words which are specific to mathematics and not usually encountered in everyday language (e.g. hypotenuse, coefficient)

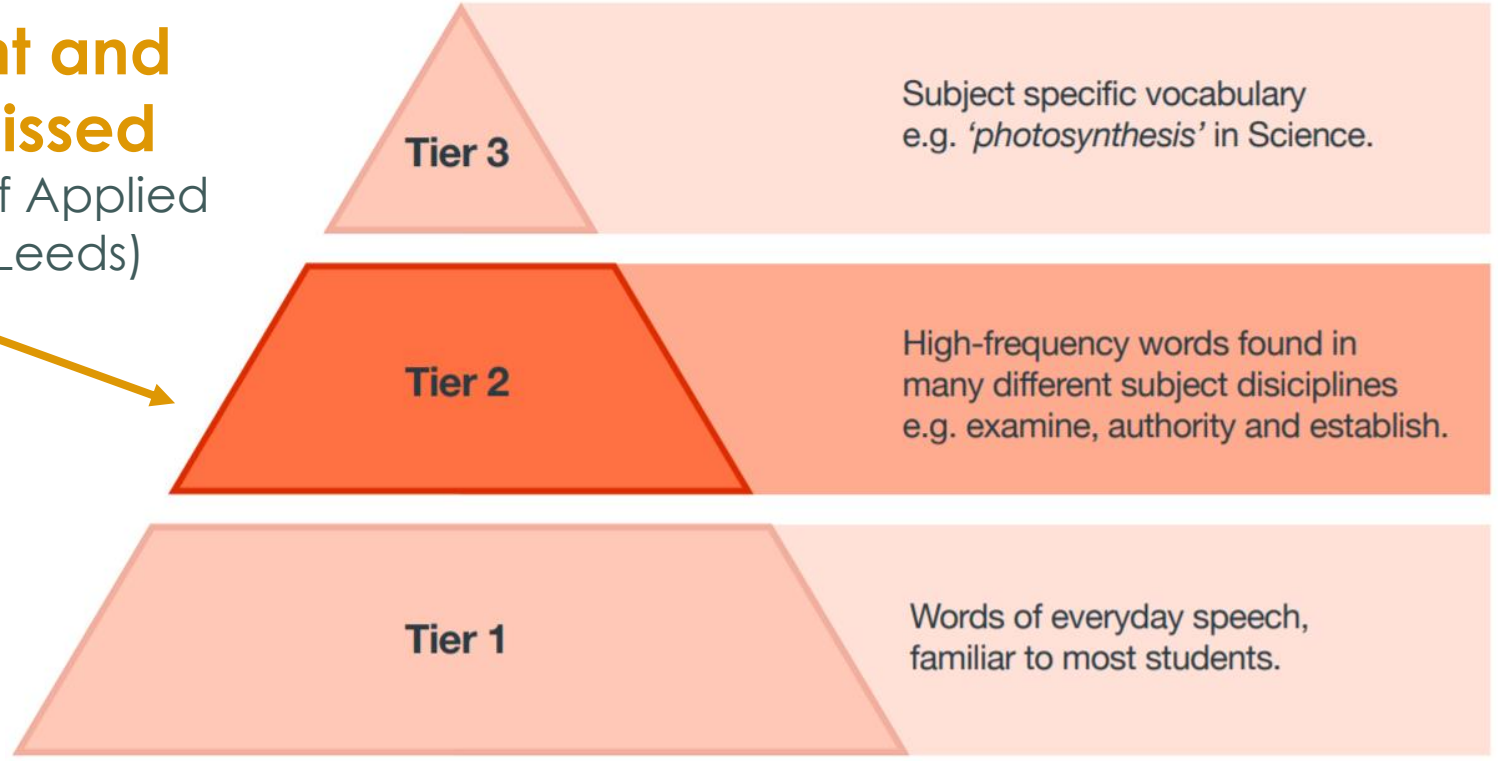
Andrew Rothbury (researcher)

Task:

- Think of five Tier 2 words, and five Tier 3 words
- How confident are you about the tiers?

Most important and likely to be missed

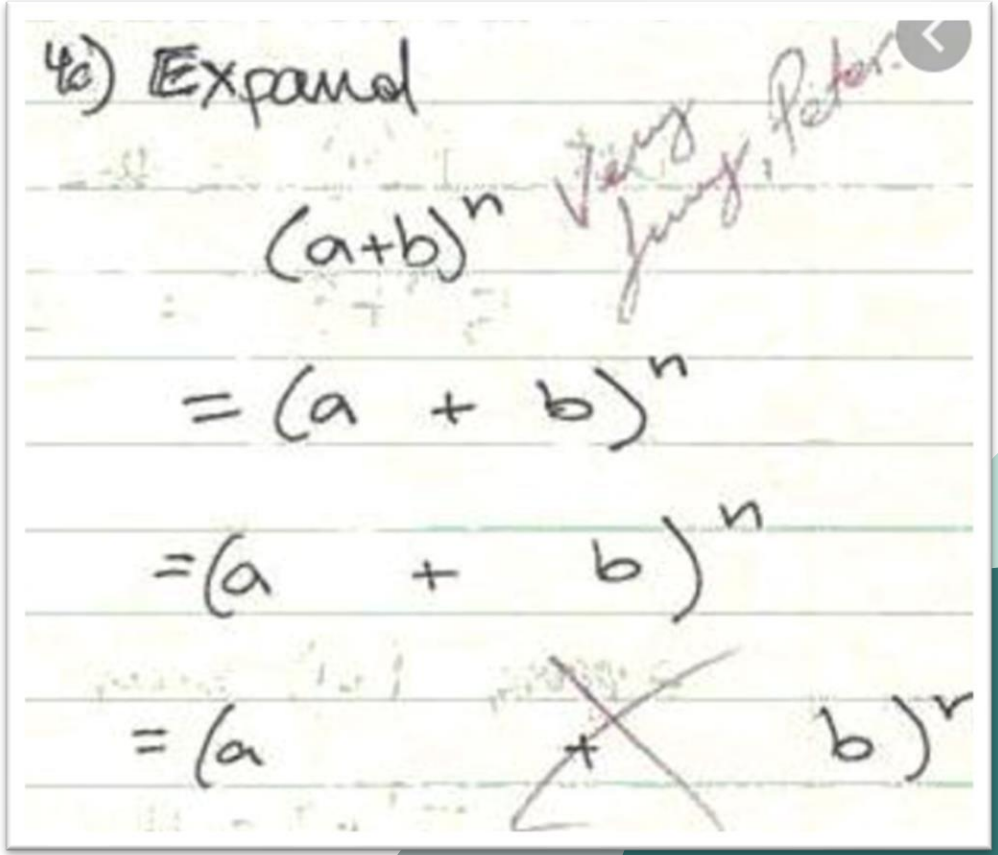
Prof Deignan (Prof of Applied Linguistics, Uni of Leeds)



Polysemous words

- Same spelling but different meanings deriving from a common origin

- How many do you have on your list?



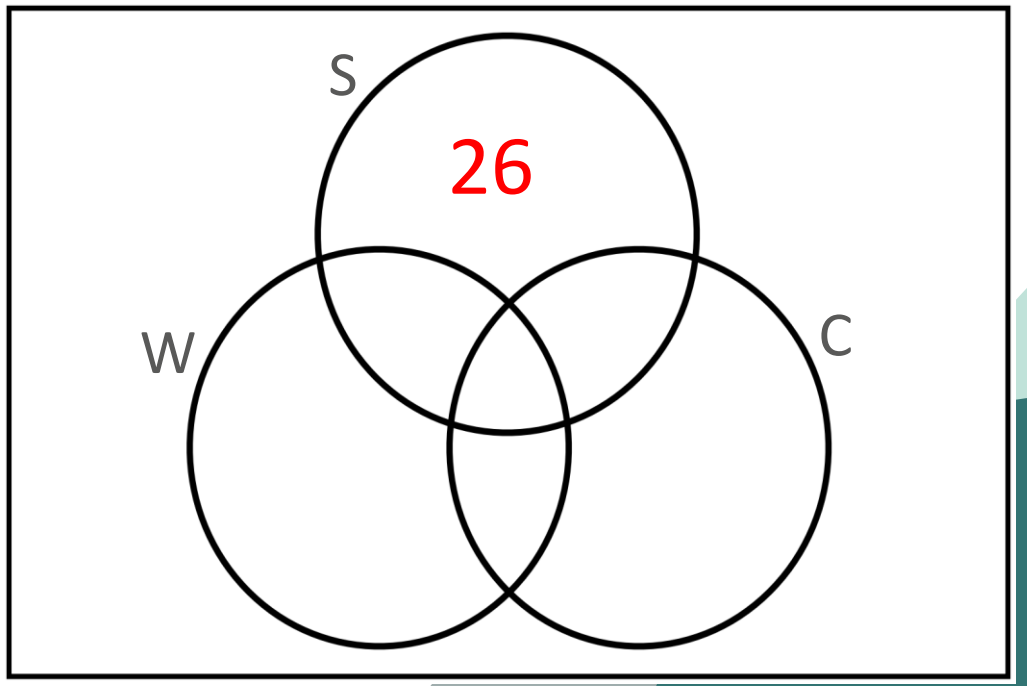
Precise, logical language

- Venn diagram questions

A group of 100 people gave the following information about the types of cheese they liked.

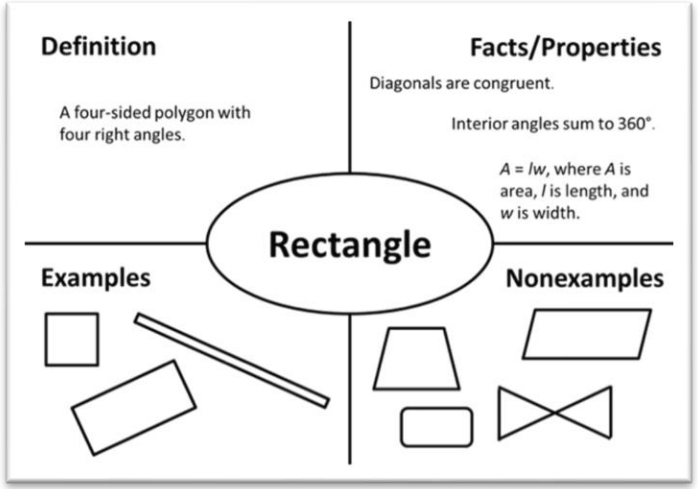
26 liked Stilton
43 liked Wensleydale
40 liked Cheddar
17 liked Stilton and Cheddar
29 liked Wensleydale and Cheddar
16 liked Wensleydale and Stilton
12 liked all three cheeses.

Draw a Venn Diagram to show this information.



What can help?

Frayer models / checklists



Etymology (origin)

hexagon (n.) 1560s, from Latin hexagonum, from Greek hexagonon, from hex "six" + gonia "angle" (see knee)

bi, di, duo	two	biweekly
circum	around	circumference
centi	hundred	centimeter
cide	kill	homicide
dec	ten	decade
div	separate	divide

Morphology (structure)

Collaboration opportunities

- Discuss and use vocabulary as a department

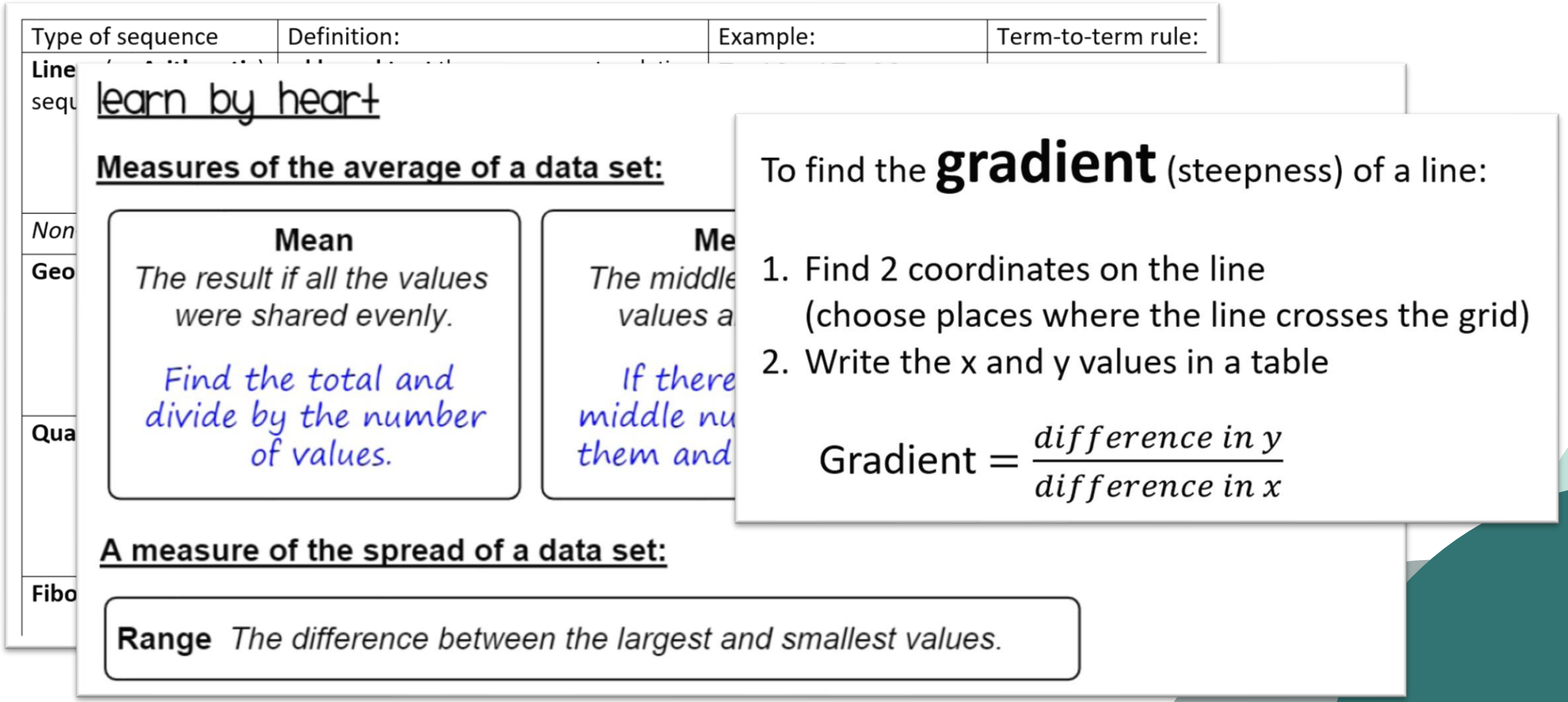
- How many of these do you know AND USE?

  - Minuend
  - Subtrahend
  - Commutative
  - Vinculum

- Knowing a word allows you to discuss! (see next section…)

Takeaways: vocab and language

- Checklists https://www.mathspad.co.uk/



Takeaways: vocab and language

- Maths Dictionary https://www.mathsisfun.com/definitions/index.html

Definition of

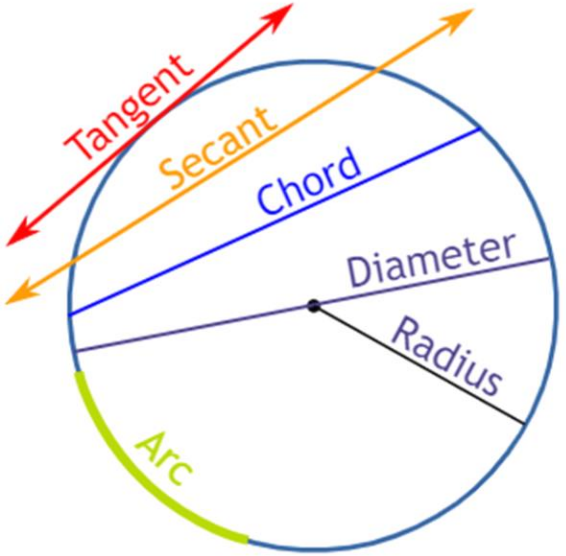
Diameter

more ...

The distance from one point on a circle **through the center** to another point on the circle.

It is also the longest distance across the circle.

And it is twice the radius.



Coordinates

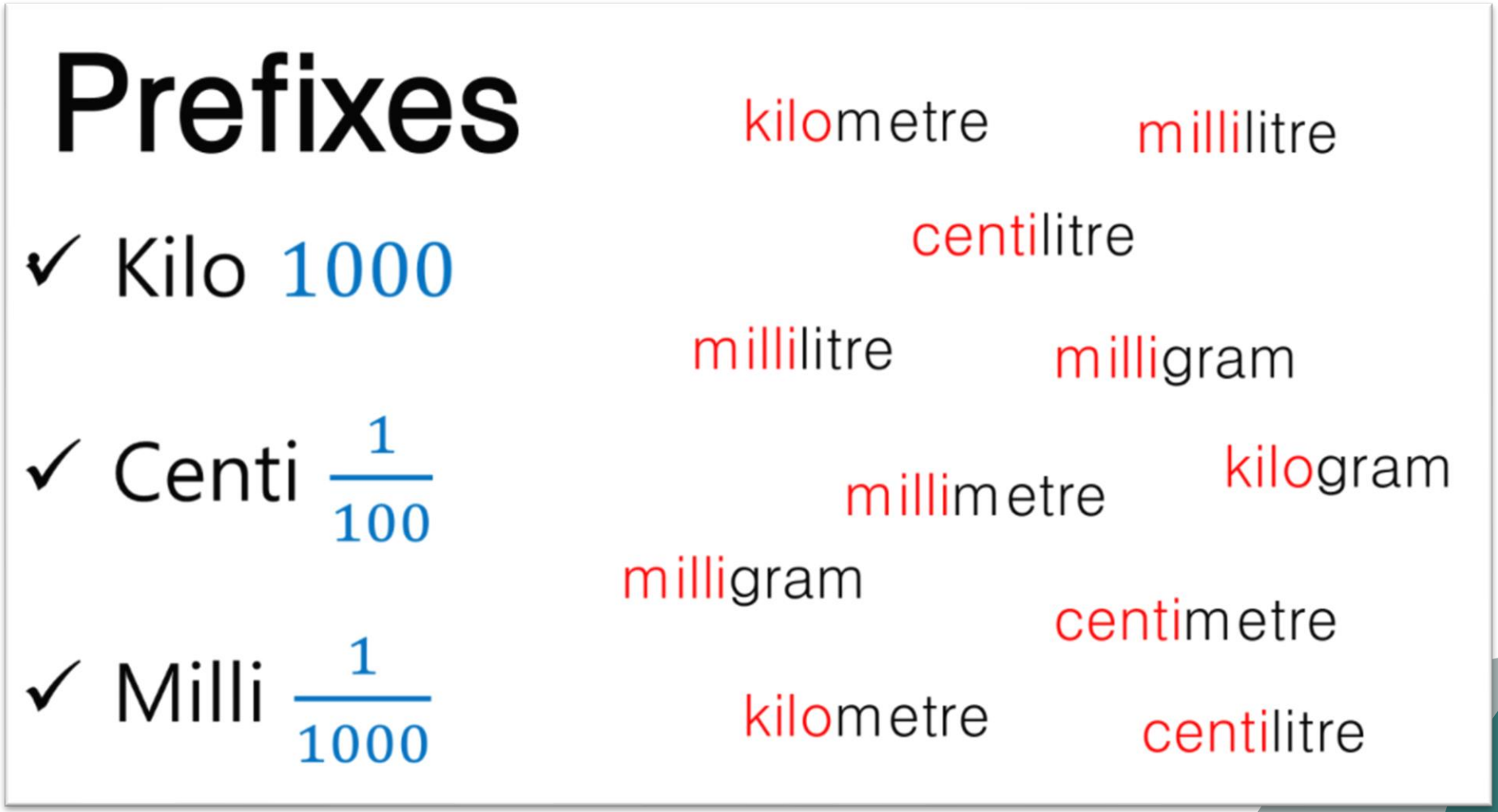
Radius (polygon)

Remainder

RMS

Takeaways: vocab and language

- Jo Morgan https://www.resourceaholic.com/p/topics-in-depth.html



Takeaways: vocab and language

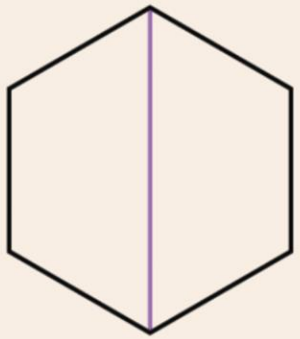
- BossMaths https://www.bossmaths.com/vocab/

διά (dia) *prep.* across (Ancient Greek)

diagonal *adj.*

1. In geometry, joining two non-adjacent vertices.
2. In everyday English, slanted.

See also: *gonia* (Ancient Greek) meaning *angle*.

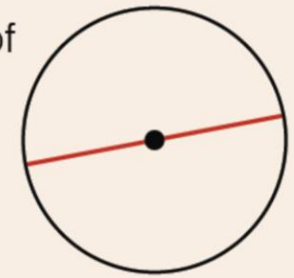


Here is a diagonal of this hexagon.
Note that it is does not appear slanted in this orientation.

diameter *n.*

1. Any straight line between two points on the circumference of a circle that passes through the centre of the circle.
2. The length of such a line.

Here is a diameter of the circle.



Comprehension and understanding

Comprehension and understanding

From Edexcel C3 2013

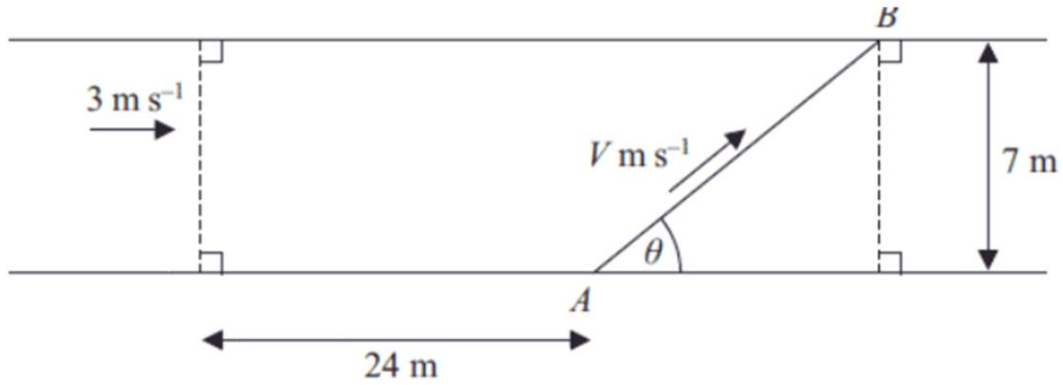


Figure 2

Kate crosses a road, of constant width 7 m, in order to take a photograph of a marathon runner, John, approaching at 3 m s^{-1}.
Kate is 24 m ahead of John when she starts to cross the road from the fixed point A.
John passes her as she reaches the other side of the road at a variable point B, as shown in Figure 2.
Kate's speed is V m s^{-1} and she moves in a straight line, which makes an angle θ, $0 < \theta < 150°$, with the edge of the road, as shown in Figure 2.

You may assume that V is given by the formula

$$V = \frac{21}{24\sin\theta + 7\cos\theta}, \qquad 0 < \theta < 150°$$

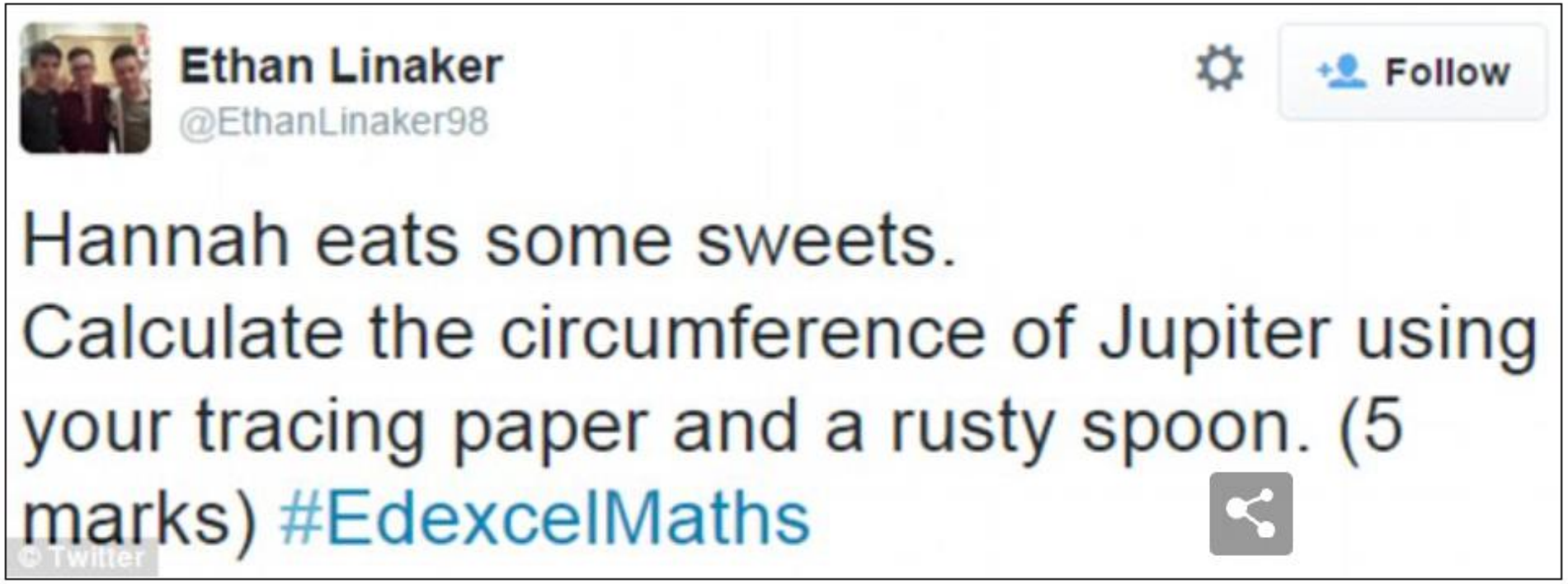
(a) Express $24\sin\theta + 7\cos\theta$ in the form $R\cos(\theta - \alpha)$, where R and α are constants and where $R > 0$ and $0 < \alpha < 90°$, giving the value of α to 2 decimal places.

(3)

Comprehension and understanding

From Edexcel C3 2013

19 There are n sweets in a bag.



(a) Show that $n^2 - n - 90 = 0$

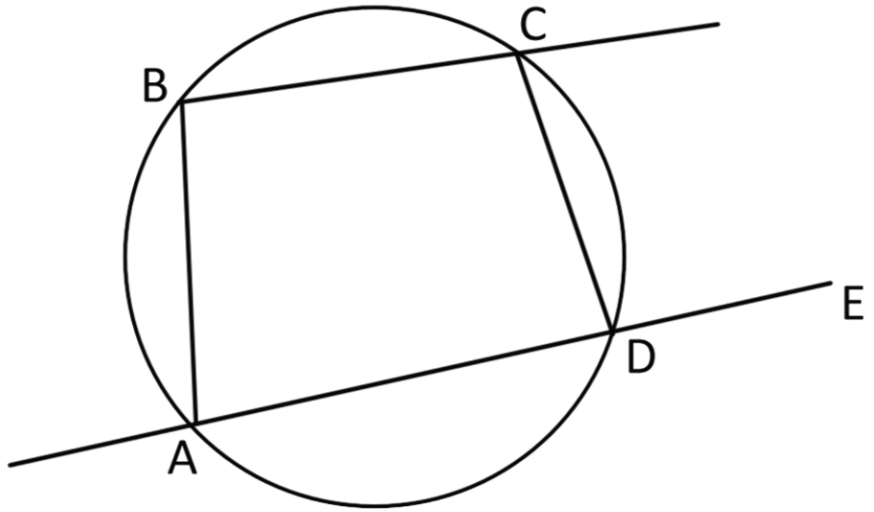
What does our ideal student do?

- Uses multiple representations
- Decodes complex instructions and sentences
- Picks out the pertinent information
- Knows when context is important and when it is not
- Makes mathematical models of situations
- Does not always read from left to right

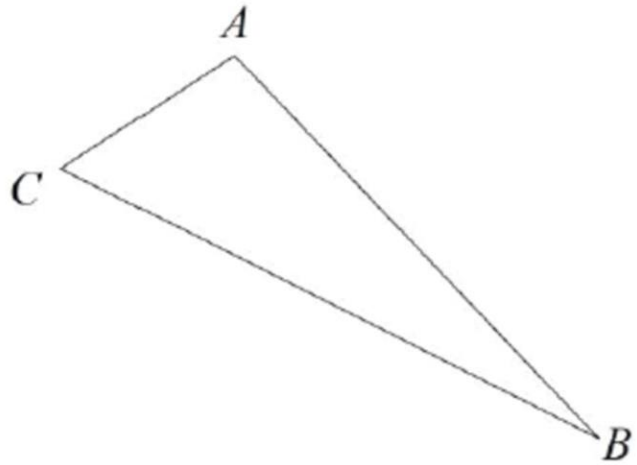
What are the difficulties?

- Using words in conjunction with diagrams

ABCD is a trapezium. ∠CDE = 98°. Find ∠BAD.



(3) Triangle ABC is shown in the diagram below.



$AC = x$
$BC = 3x$
Angle $ACB = 60°$

Show that the perimeter of the triangle is $\left(4+\sqrt{7}\right)x$.

What are the difficulties?

- Using words in conjunction with diagrams
- Unpicking the maths from the context
- Making assumptions:

  “it looks like a square therefore it is a square”

What are the difficulties?

- Multiple reading directions (Nolan, 1984)

2456	113.006	$\begin{array}{r} 10 \\ -\ 6 \\ \hline 4 \end{array}$		$\frac{2x+1}{x}+\frac{7}{x+1}$	
Left to right	Right to left	Top to bottom	Bottom to top	Diagonal	Circular

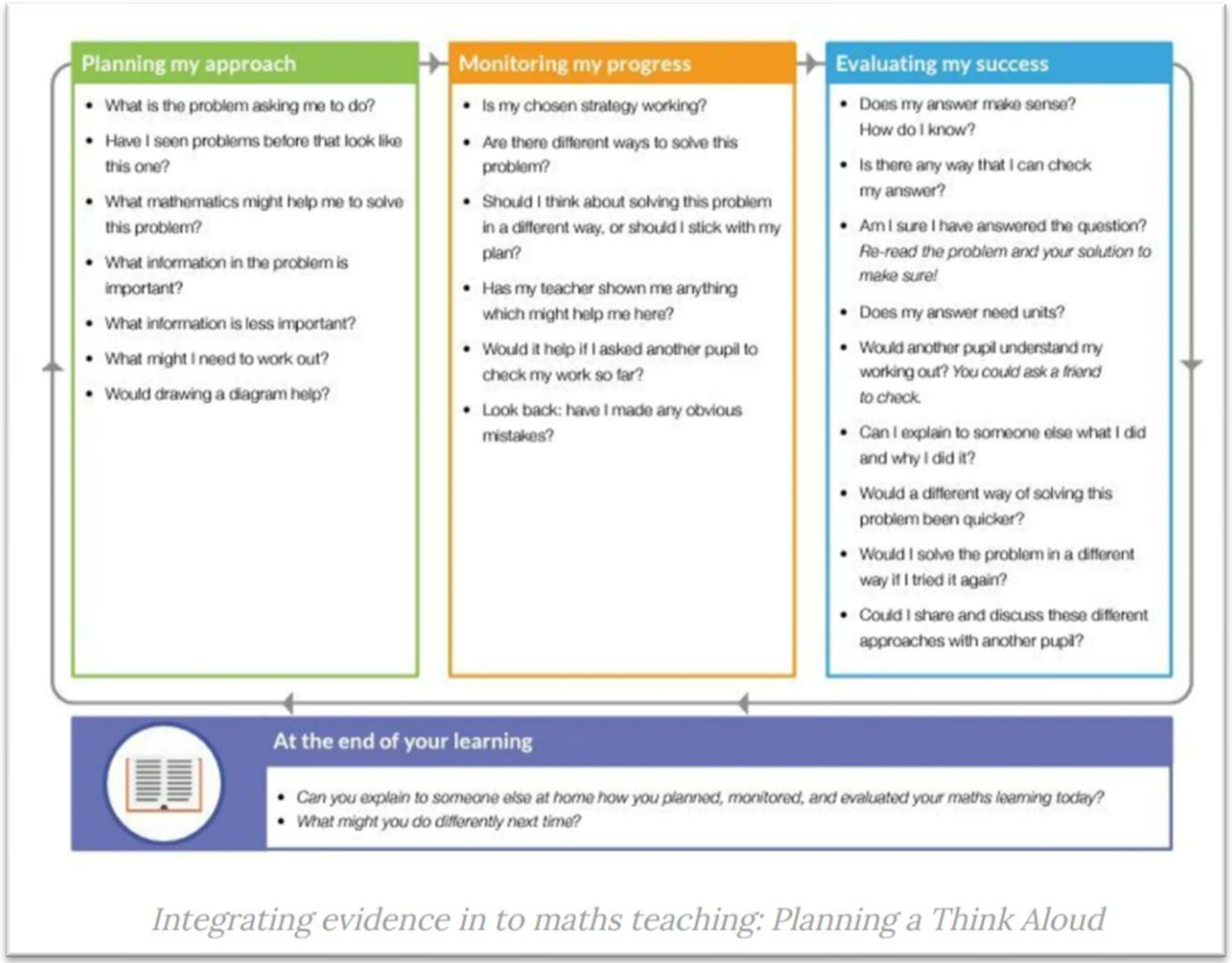
What can help?

- Relating words to diagrams and using multiple representations
- Teaching specific definitions and overlearning
- **Micro rules**
- **Metacognition: thinking out loud**

- Modelling and worked examples
- Spotting errors in someone else's work

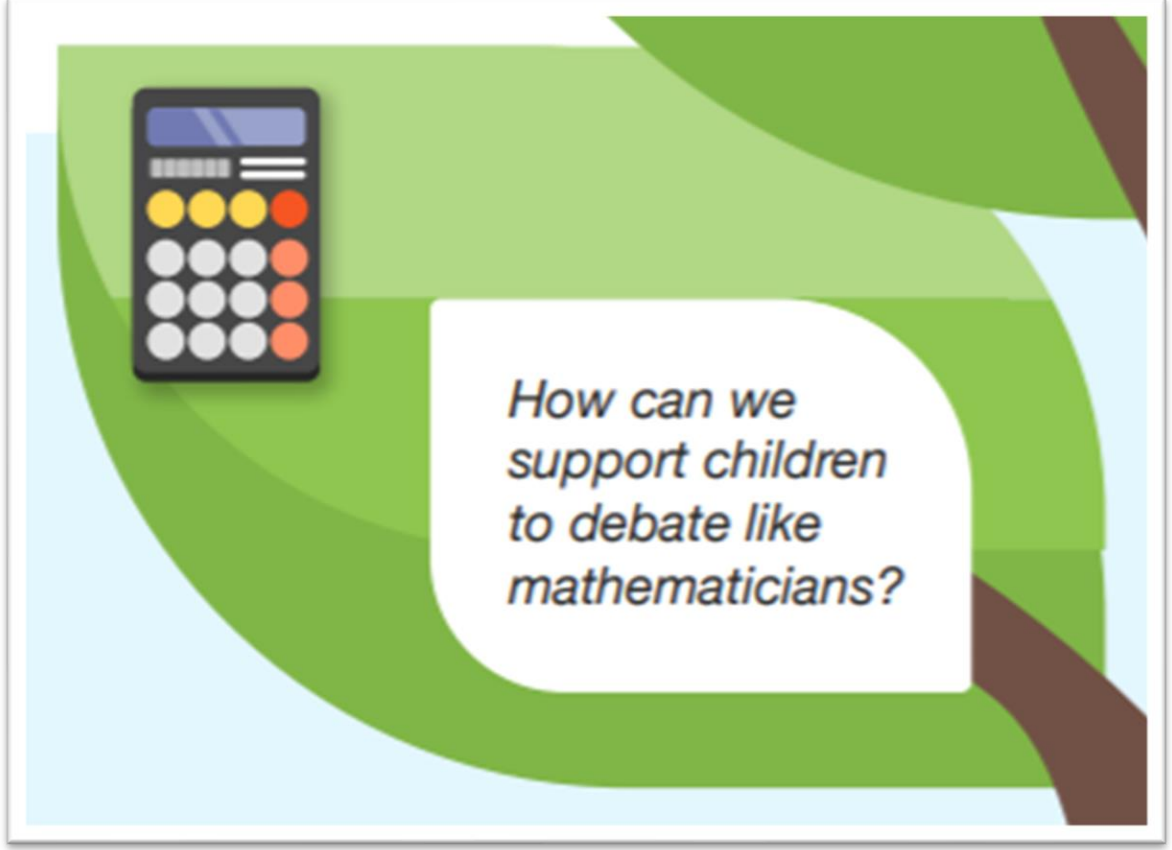
Takeaways: Comprehension

- Think Alouds (EEF Blogs – Kirstin Mulholland, Emma Barker)



Talking mathematically

Talking mathematically



What does our ideal student do?

- "debate like a mathematician" (EEF Guidelines)
- Reason and justify
- Give full solutions, not just answers
- Use **mathematically correct language**

"can you say it better?"

Departmental focus?

What are the difficulties?

- Dumbing down
  "Commutative law of multiplication"
- Teachers being sloppy with language
  "angles on a straight line add to 180"
- Asking closed questions
- Lack of confidence

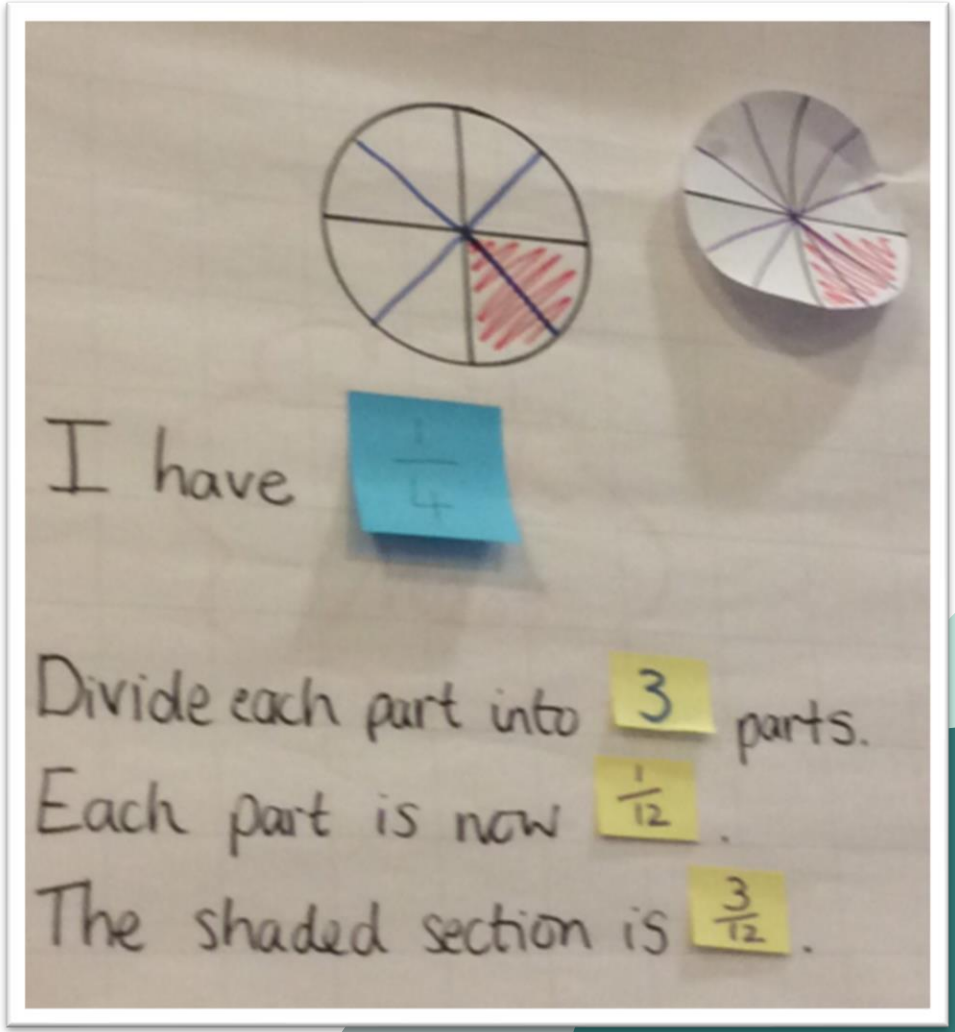
What can help?

- Teach the proper language and definitions: **Agree on these!**
- Use Socratic questioning
- Think – pair – share – warm call
- Insist on full sentance answers

…asking and answering questions to stimulate critical thinking and to draw out ideas and underlying presuppositions

Takeaways: stem sentences

- Maintain focus
- Recognising what's important
- Reducing cognitive load
- Connecting ideas
- Using correct language
- Providing high quality shared language

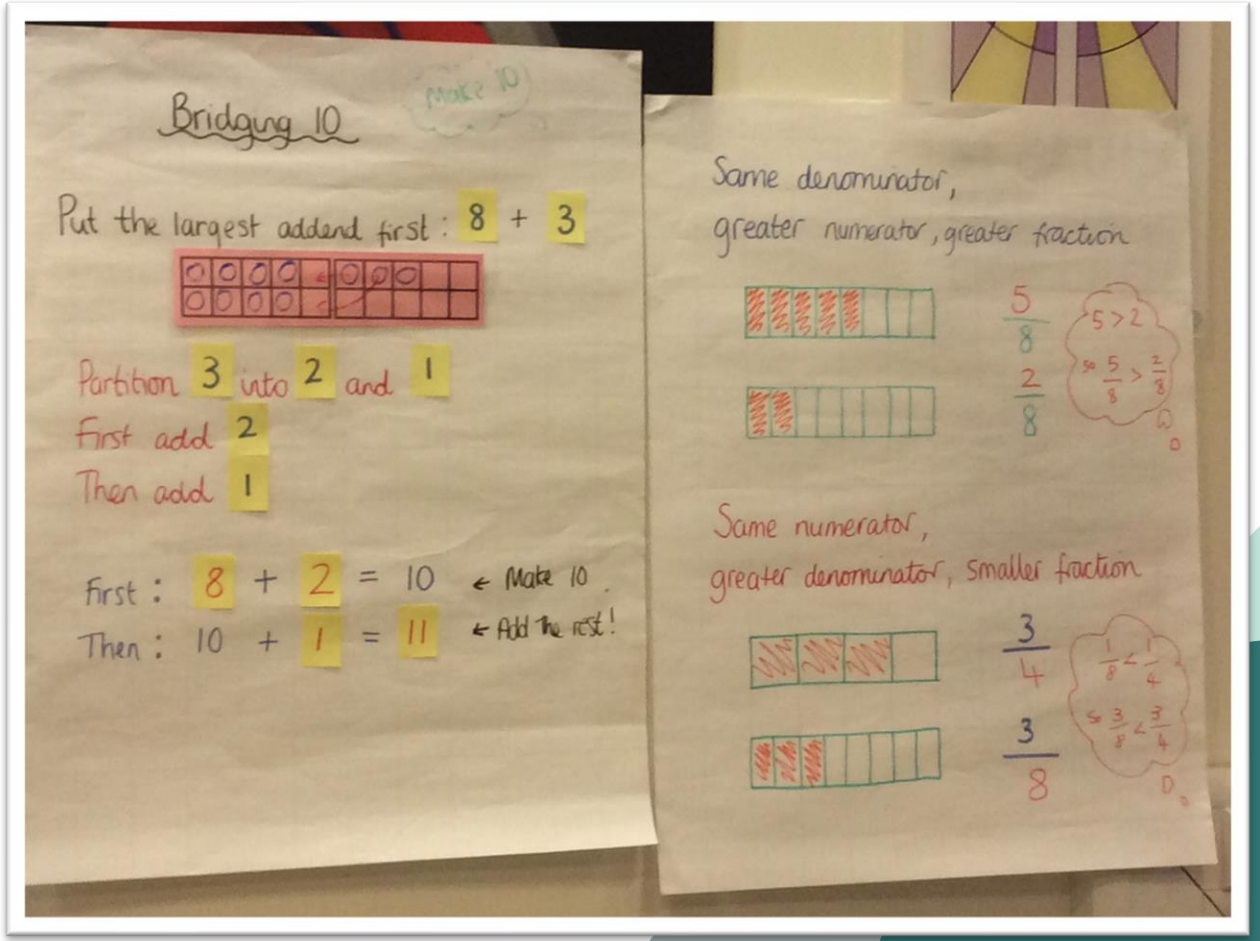


Stem sentences vs generalisations

- Not definitions
- Mathematically accurate
- Supported by images

- Drawn upon continually

"the multiplicative relationship between __ and __ is __"



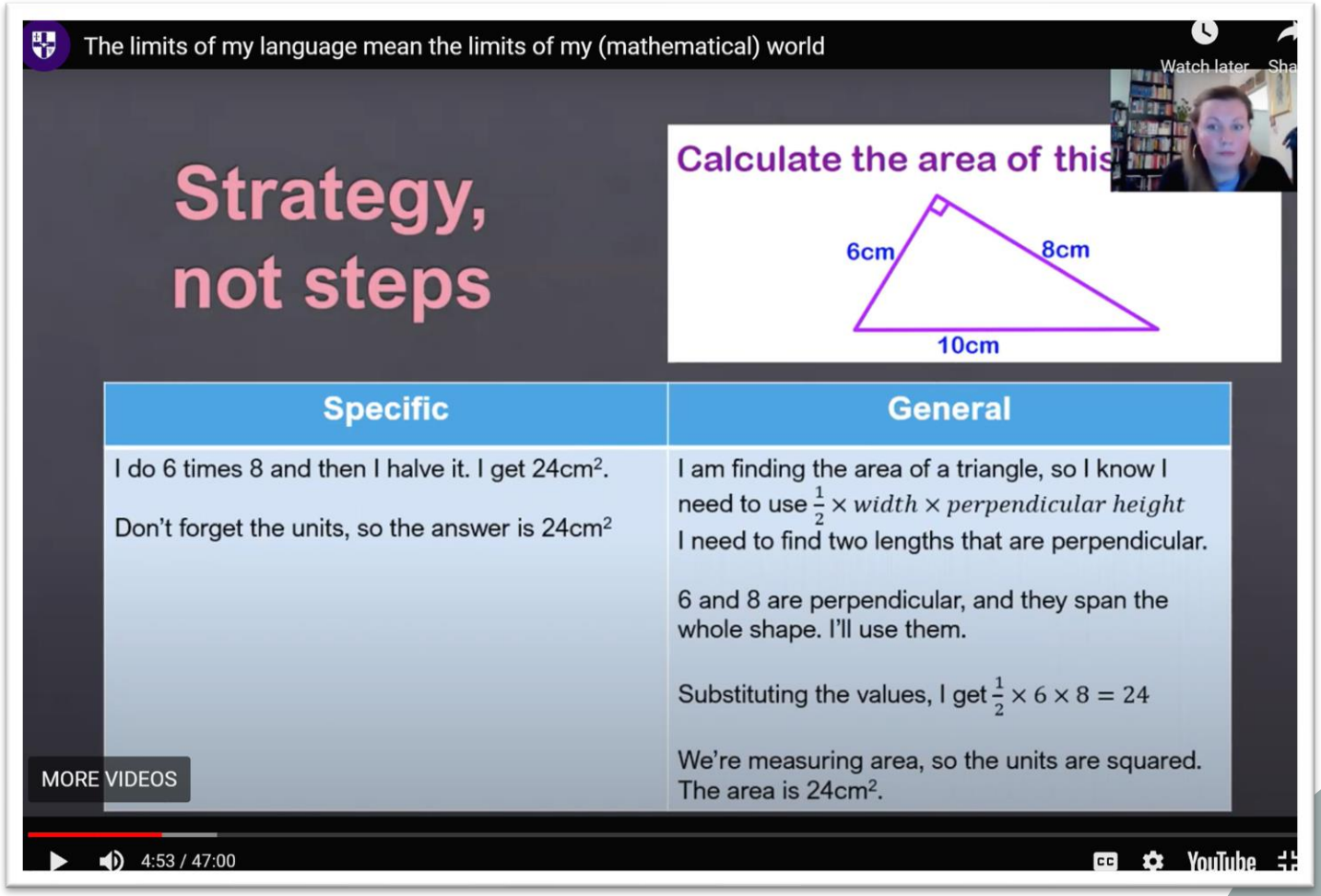
Takeaways: directed numbers

	adjective	noun	verb
+	positive	plus	to add
-	negative	minus	to subtract

(-3) – (-4)=

Takeaways: talking mathematically

- Dani Quinn https://www.lboro.ac.uk/services/lumen/professional-development/language-in-maths/



@mathsmrgordon

Increasing rigour in the maths classroom

Dani Quinn, Head of mathematics at Michaela Community School, Wembley, wrote a blog called "What Are You Thinking?" on some ways to increase rigour in the maths class room with teacher and student explanations. Here is a visual summary of the key threads:

TALK IN GENERAL TERMS

A lot of the explanations we give to pupils focus too much on the specific example and its values (e.g. 3cm, or y = 6) and not on the general underlying structure.

"To get the perimeter add 6 and 7 and 6 and 7 and don't forget the units."

"To get the perimeter I need to add the lengths around the outside. Some of them are blank: I will fill them in. I know that opposite lengths in a rectangle are equal; that is why these lengths are also 6 and 7. I will now add all the lengths. The lengths are measured in millimetres, so I will give the answer in millimetres."

CLAMP DOWN ON PRONOUNS

Watch out for when you – and more often the pupils – use pronouns to disguise uncertainty and wishy-washy thinking. This is usually 'it', 'they' and 'them.'

For example:
Change "*You multiply them*" to "*Multiply the base and the perpendicular height*"
Change "*You find its height*" to "*Find the slant height of the parallelogram*"
Change "*You add them and divide by how many there are*" to "*Add the values and divide by the number of values*"

USE ELABORATIVE INTERROGATION

A lot of the questions in maths classrooms have a 50/50 chance of being correct, which means we are probably getting a lot of false positives, especially considering that our body language often conspires to give away the 'correct' answer.

For example
Change "Are these lines parallel?" to "How do you know these lines are parallel?"
Change "Which number should I use for height, 8 or 6?" to "How do we know which of the numbers to use as the height in this question?"
Change "What is the value of the angle x?" to "How do you know whether to subtract from 180 or 360?"

If the 'why' and the 'how do you know' and 'how to decide' is not a core part of your teaching and explanation, it can't possibly be part of the pupils' responses!

HELP THEM TO BUILD EXPECTATIONS

Make sure the pupils have some expectations around the answer before they get started! Similarly, teach them to use words in the question to begin visualising or imagining things.

For example

"It is an enlargement of a negative scale factor so will be drawn around **this area** (draw arrow)"

"If I sketch the circle and the straight line I can see where they might intersect"

"One of the angles in the triangle is 80 so my answer must be less than 100."

Help the children feel familiar with the process and then move to expectations as, by then, they have a few examples in their mind to test their expectations.

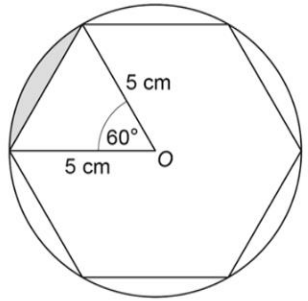
Writing mathematically

Writing mathematically

- Possibly the hardest to achieve!
- Think back to the GCSE questions

The vertices of a regular hexagon lie on a circle with centre O and radius 5 cm

Not drawn accurately



Work out the shaded area.

Give your answer in the form $\frac{a\pi - b\sqrt{c}}{12}$ where a, b and c are integers.

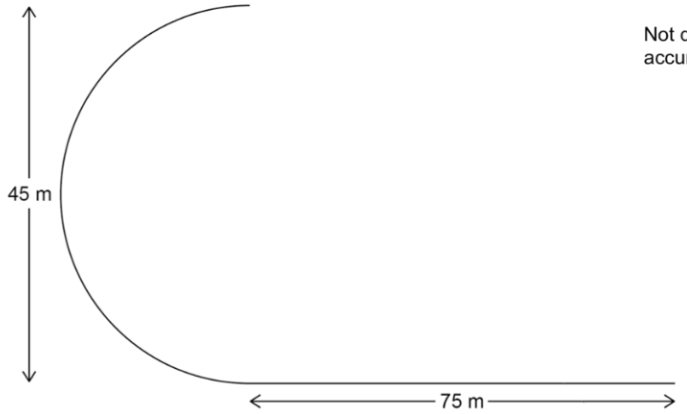
[4 marks

Part of a running track is the arc of a semicircle joined to a straight line.

The semicircle has diameter 45 metres.

The straight line has length 75 metres.

Not drawn accurately



Abby runs once along this part of the track in 18 seconds.

Work out her average speed.

Give your answer to 2 significant figures.

[4 marks

What does our ideal student do?

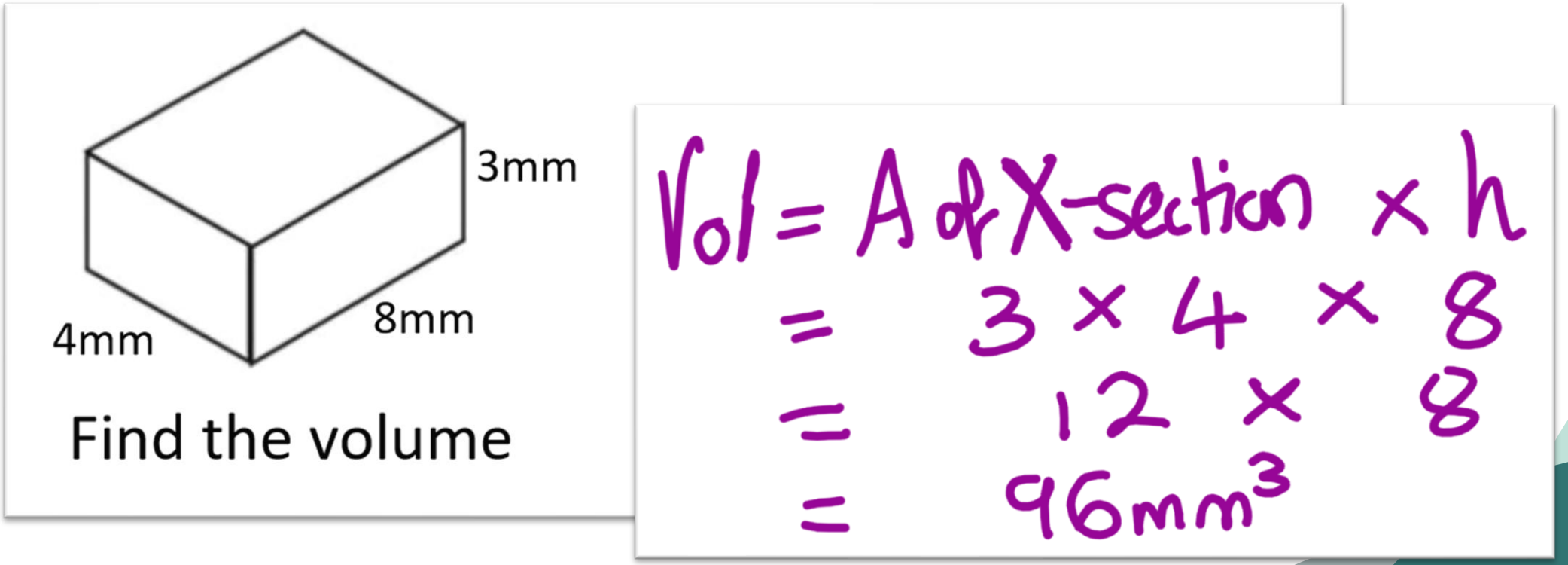
- Work downwards
- Write in “full sentences”
- Use correct mathematical notation
- Annotate when necessary
- Draw conclusions

What are the difficulties?

- Sketching: understanding these are representations not accurate scale drawings
- Understanding that the representation is important, not the actual size (eg bar models etc)
- Consider paper in exercise books: plain? Squared?

What are the difficulties?

"I know what you mean"



What can help?

- Worked examples
- No dumbing down
- Gap filling exercises
- Modelling – lots of it!
- Include multiple representations
- Using tricky numbers so working has to be shown
- Structuring a proof

Worked examples

$$3 \times 25 \times 4$$

$$\begin{aligned} & (3 \times 25) \times 4 \\ = \ & 75 \times 4 \\ = \ & 300 \end{aligned}$$

NOTE: we work DOWNWARDS and keep th
the calculation

$(3 \times 25) \times 4 =$

Simplify $6ab + 7b + 2ba - 3b$

$$= 6ab + 2ab + 7b - 3b$$

$$= (6+2)ab + (7-3)b$$

$$= 8ab + 4b$$

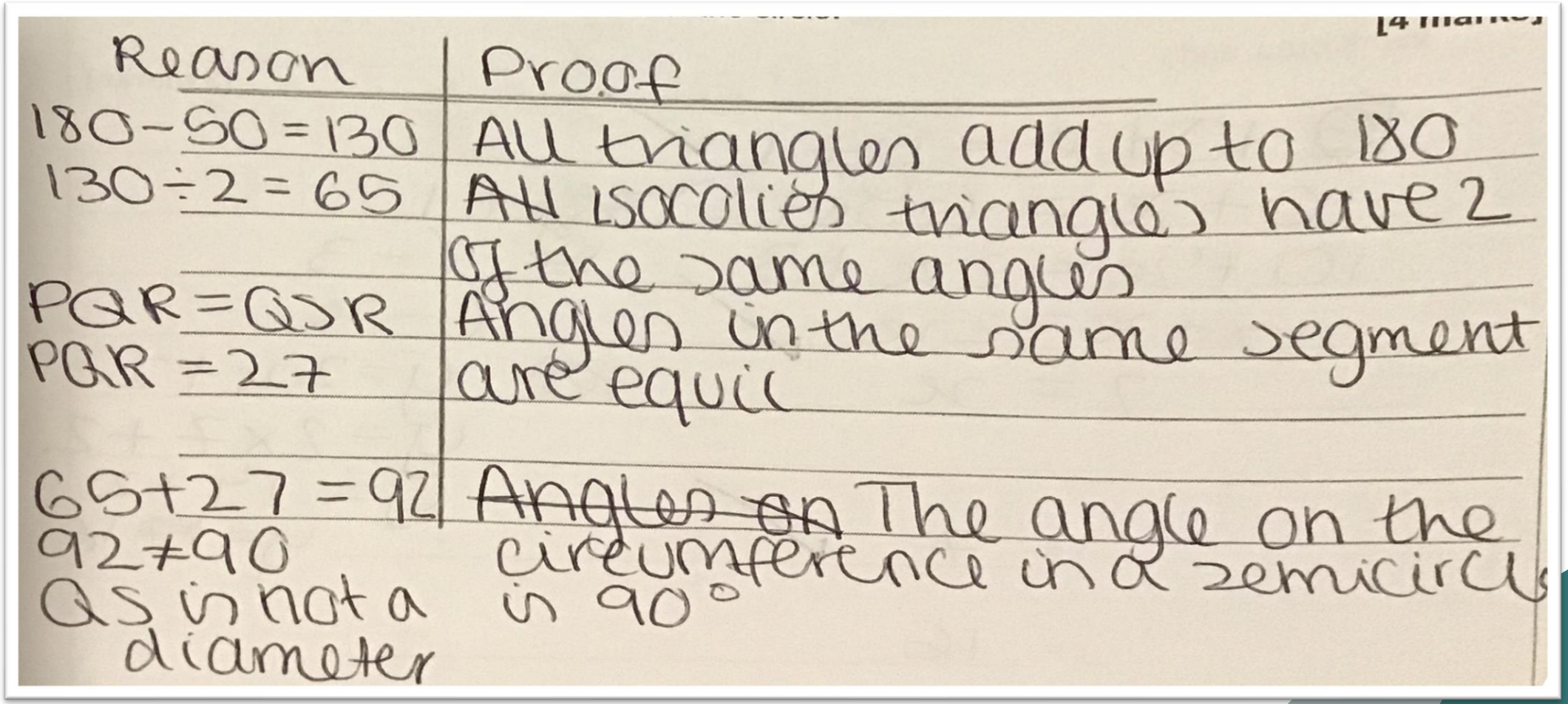
Worked examples: gap filling

Equation	Quadratic formula	Simplified	Solutions (2dp)
$x^2 + 4x + 2 = 0$	$\frac{-(\ ) \pm \sqrt{(\ )^2 - 4(\ )(\ )}}{2(\ )}$	$x = \frac{-4 \pm \sqrt{8}}{2}$	$x =$ and $x =$
$x^2 - 5x + 3 = 0$	$\frac{-(\ ) \pm \sqrt{(\ )^2 - 4(\ )(\ )}}{2(\ )}$	$x = \frac{5 \pm \sqrt{\ }}{2}$	$x =$ and $x =$
$x^2 + x \quad = 0$	$\frac{-(\ ) \pm \sqrt{(\ )^2 - 4(\ )(-3)}}{2(\ )}$	$x = \frac{\pm\sqrt{\ }}{\ }$	$x =$ and $x =$
$2x^2 \quad x \quad = 0$	$\frac{-(7) \pm \sqrt{(7)^2 - 4(\ )(1)}}{2(\ )}$	$x = \frac{\pm\sqrt{\ }}{\ }$	$x =$ and $x =$
$x^2 \quad x \quad = 0$	$\frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(-4)}}{2(3)}$	$x = \frac{\pm\sqrt{\ }}{\ }$	$x =$ and $x =$
$x^2 \quad x \quad = 0$	$\frac{-(\ ) \pm \sqrt{(\ )^2 - 4(\ )(\ )}}{2(\ )}$	$x = \frac{-3 \pm \sqrt{5}}{2}$	$x =$ and $x =$

@andylutwyche

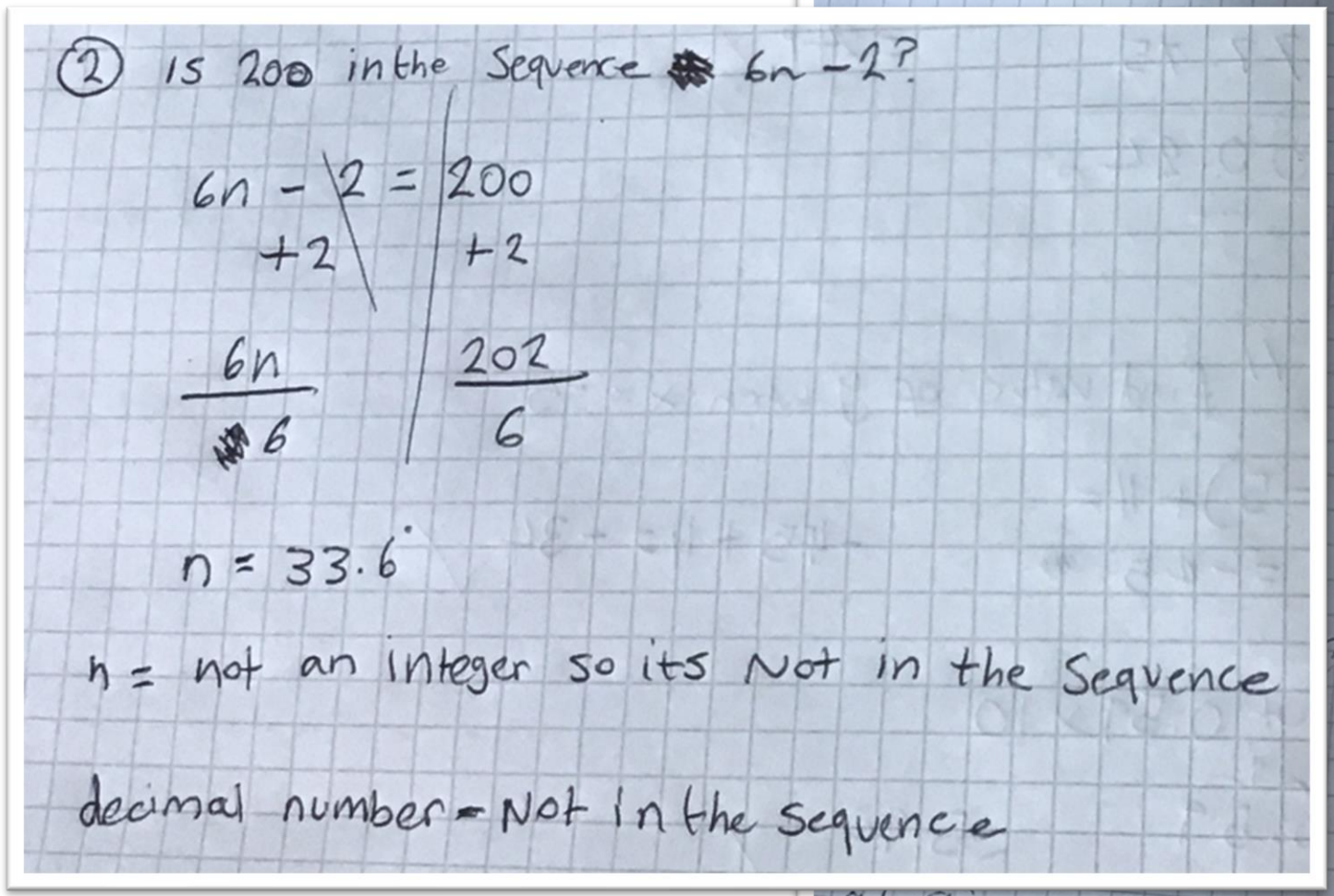
Worked examples: proof

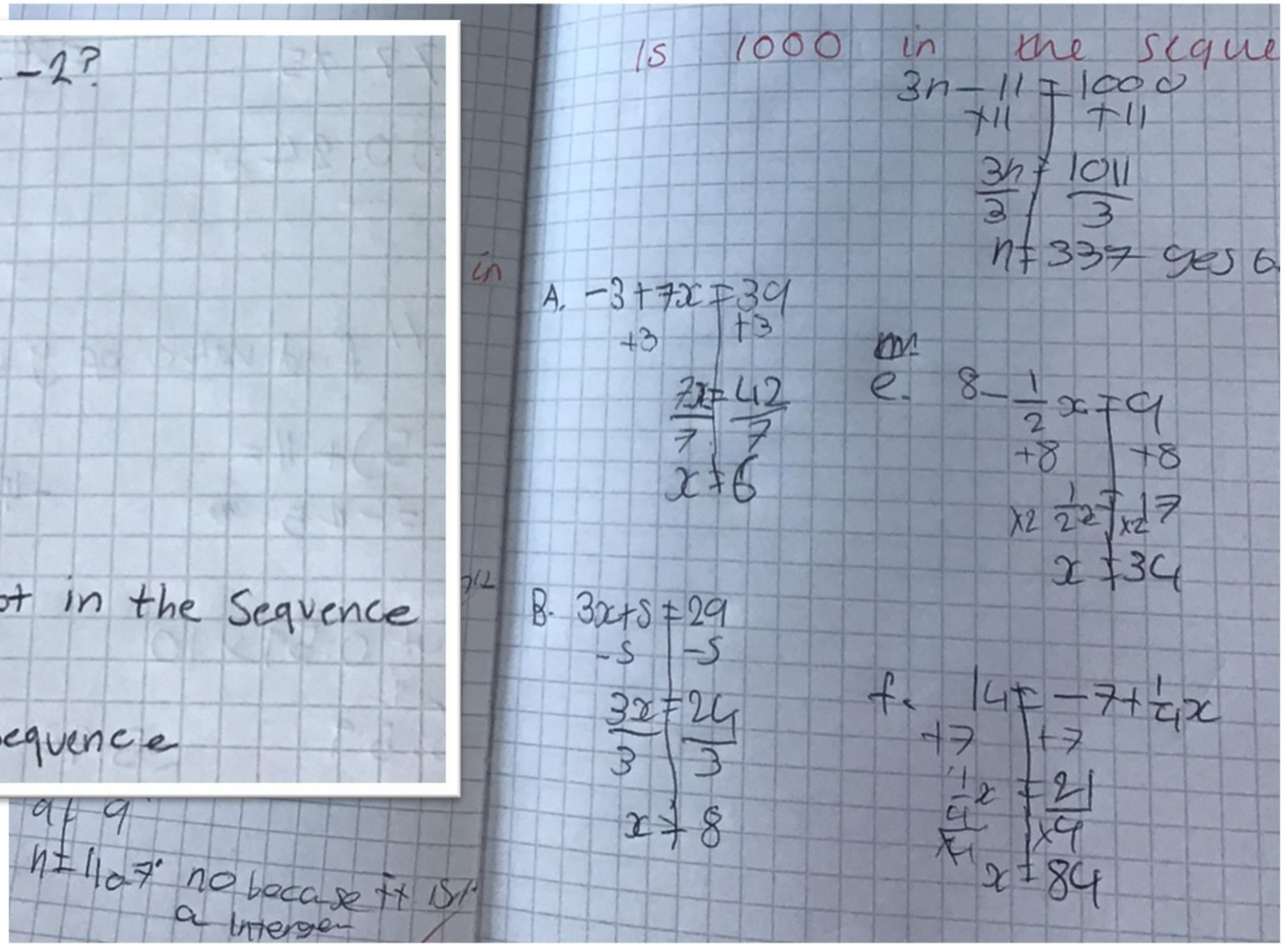
https://topdrawer.aamt.edu.au/



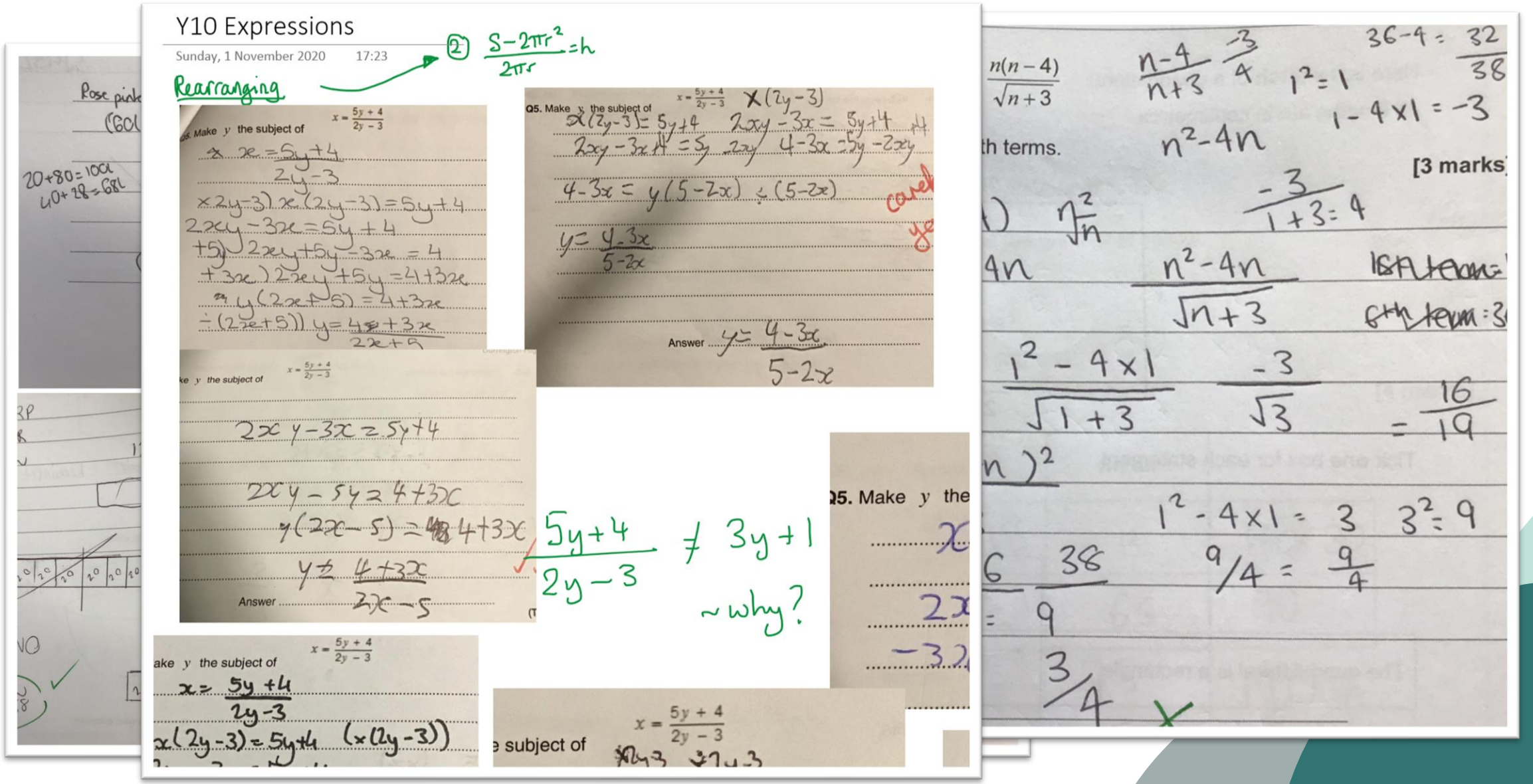
Reason	Proof
180 − 50 = 130	All triangles add up to 180
130 ÷ 2 = 65	All isocolies triangles have 2 of the same angles
PQR = QSR	Angles in the same segment
PQR = 27	are equil
65 + 27 = 92	~~Angles on~~ The angle on the
92 ≠ 90	circumference in a semicirc
QS is not a diameter	is 90°

Foundation year 10: no dumbing down

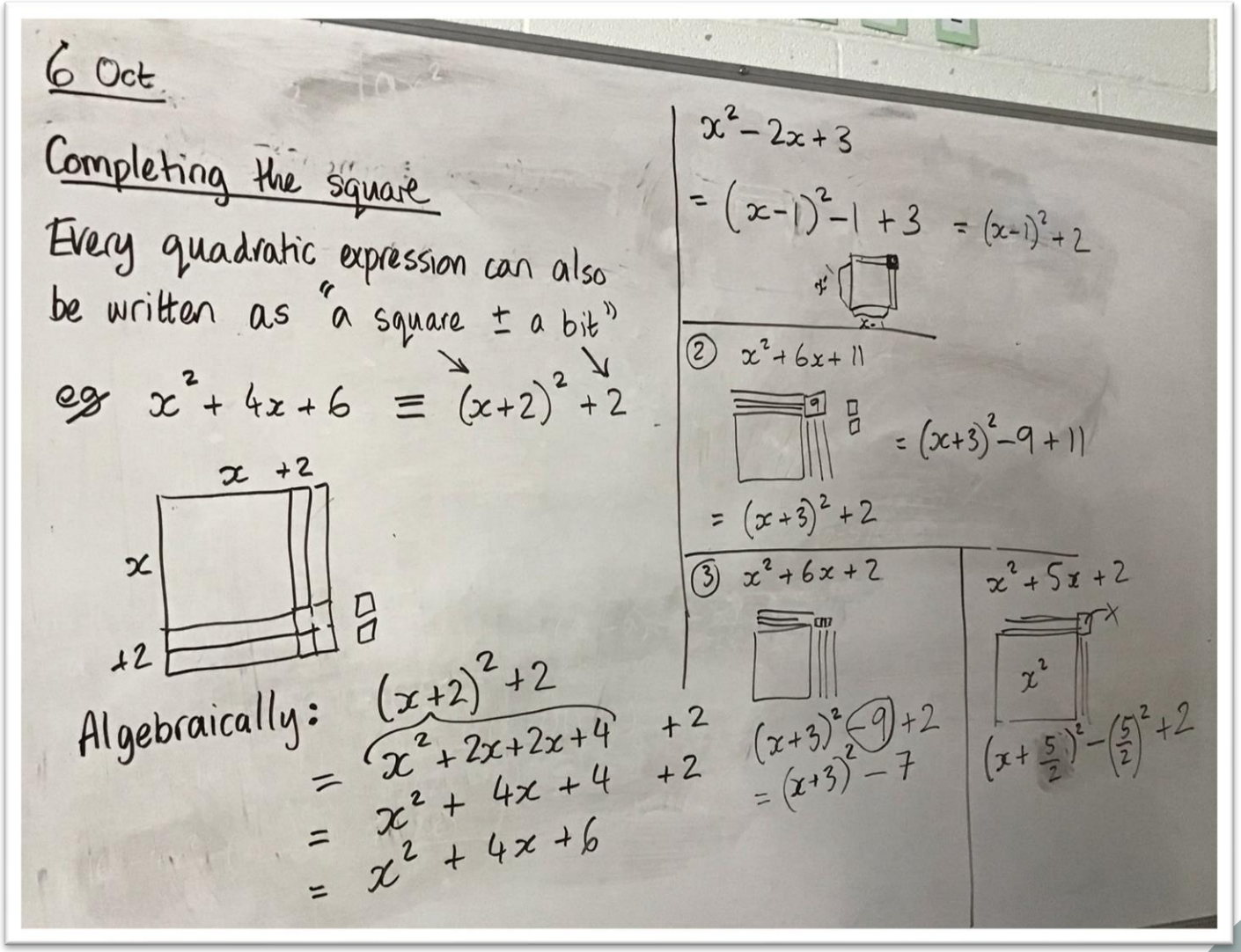




Showing work



Working board



Takeaways: writing mathematically

- MODEL MODEL MODEL
- PRAISE PRAISE PRAISE
- Work together to be **consistent** across your department

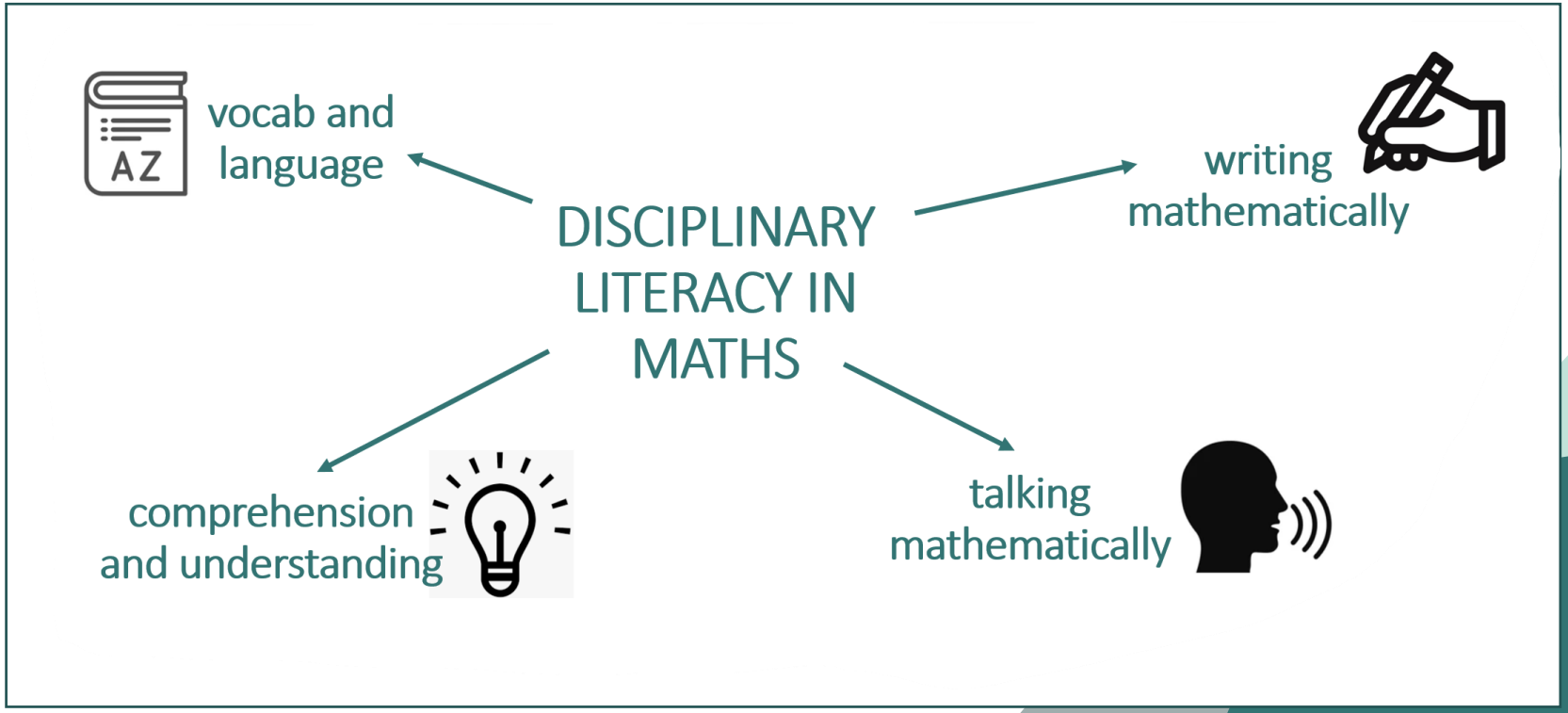
- Let's get EMOTIONS back into maths:
  - Elegance
  - Efficiency
  - "Nice" solutions

Thank you!

@runningstitch

@SEMathsHub

@DurringResearch



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