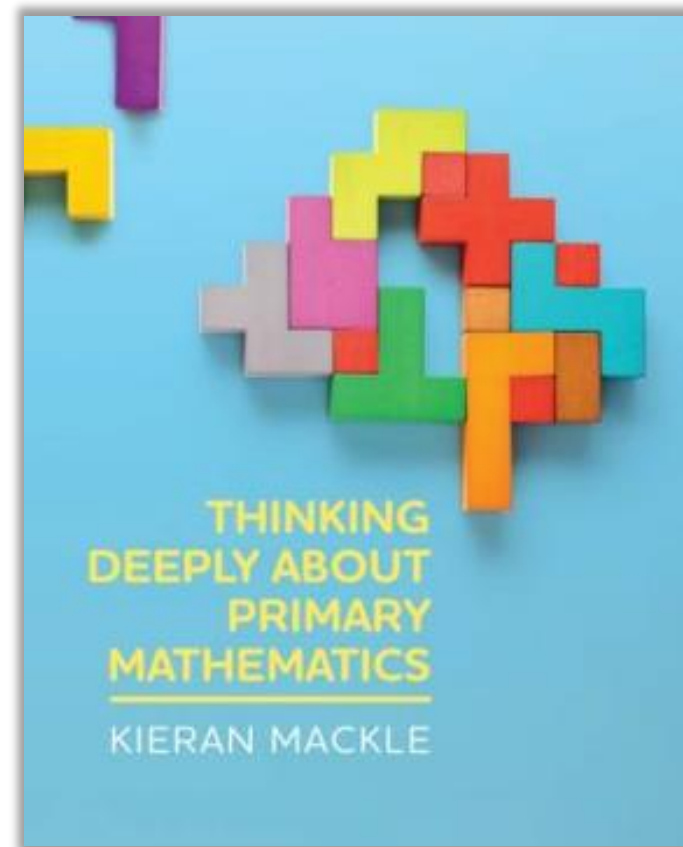
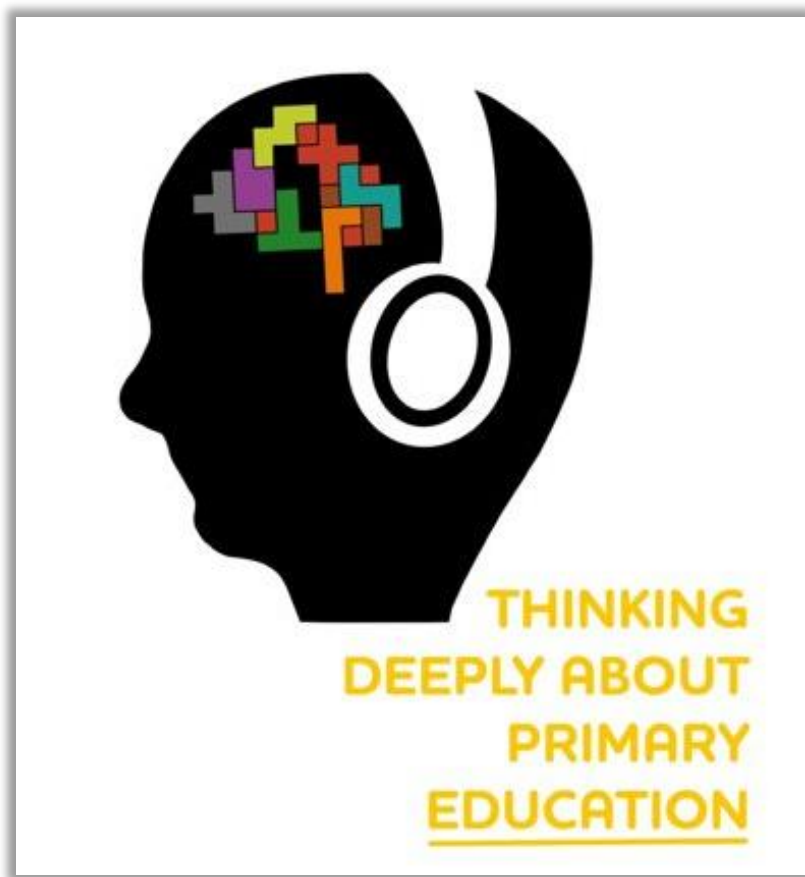


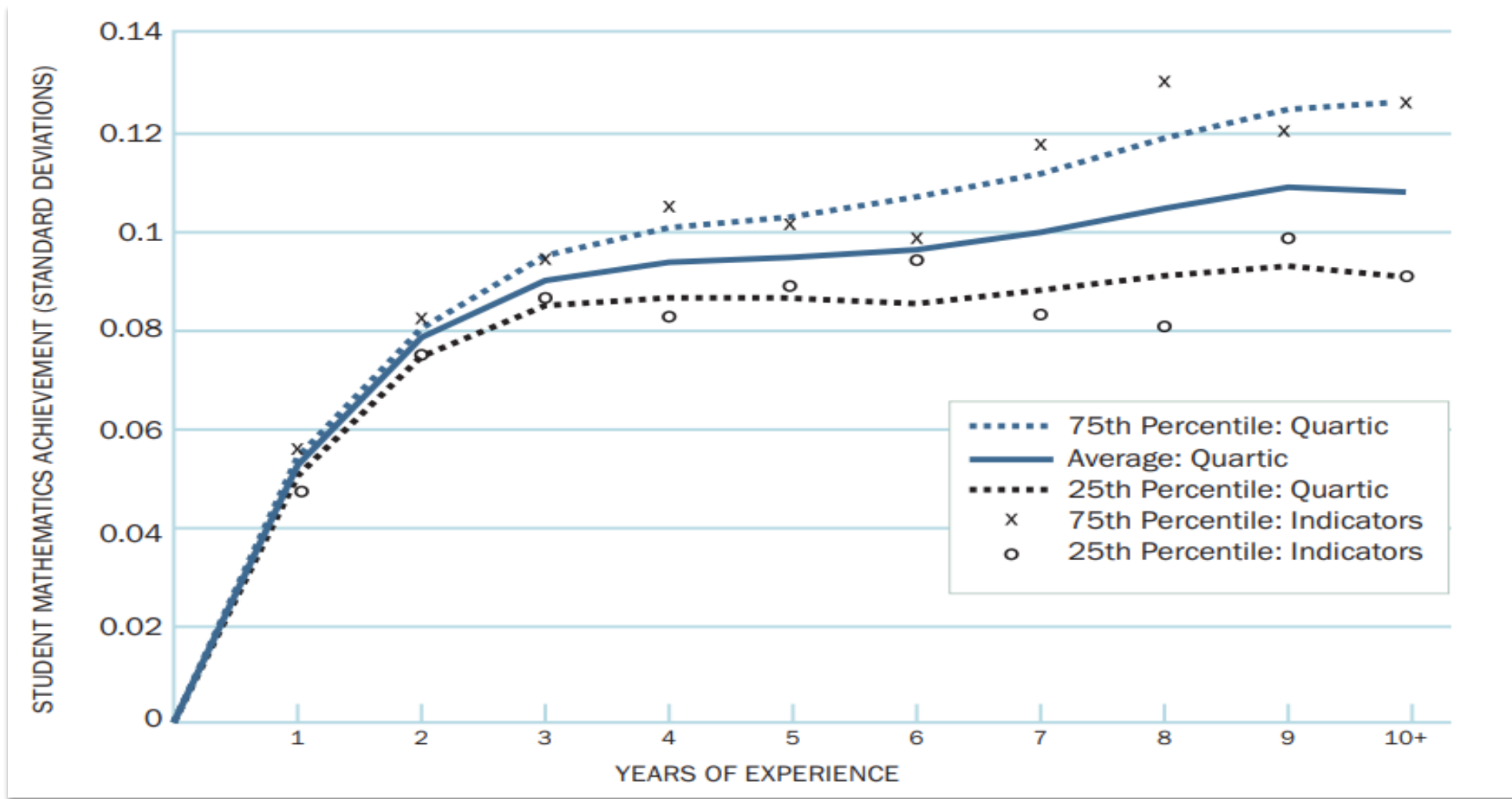
# Complete Mathematics CPD

## Thinking Deeply about Primary Mathematics

[kieran.mackle@completemaths.com](mailto:kieran.mackle@completemaths.com)

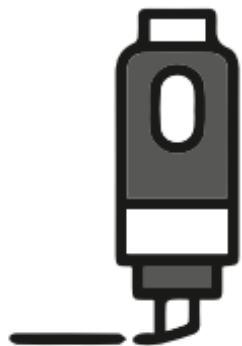
@Kieran\_M\_Ed







Highlights quality,  
making success  
apparent



Formalises measures  
of success



Designed with  
objectivity in mind

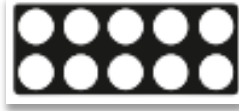


Reflects how teachers'  
practice develops  
over time

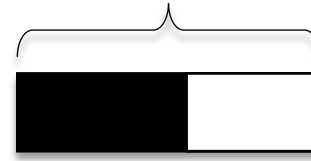




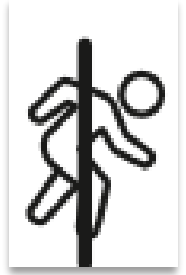
High Quality Textbooks



Models and Images



Bar Modelling



Threshold Concepts



Misconceptions



Variation Pedagogy



Instruction



Language



Reasoning



Storytelling



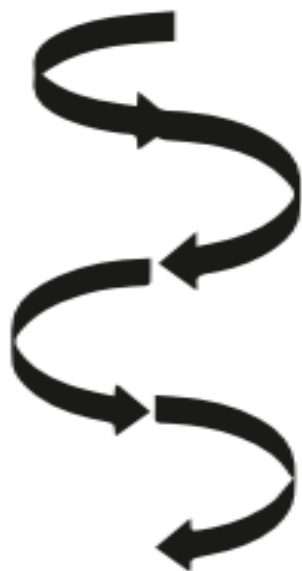
Planning



Reading for PD



Carefully sequence progression.



Stop looking at your watch.



Build challenge, problem solving, reasoning into learning episodes

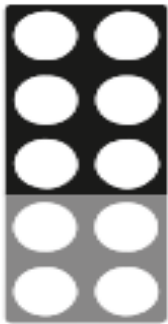


Take time to think about models, images, strategies, alternative methods





Represent inherent mathematical structures



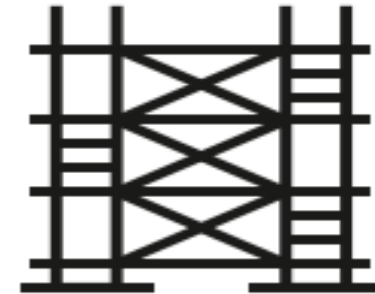
Bring clarity to explanation and instruction



Deploy fairly amongst pupils, even in the most complex of scenarios



Phase out, in the same way a scaffold comes down when the building is secure



**Cindy has 55 balloons. Kate has 44 balloons.  
How many balloons do they have altogether?**

**Cindy has 55 balloons. Kate gives her 44 balloons.  
How many balloons does Cindy have now?**



The Kölntriangle is 103m tall.  
LVA Hauptgebäude is 123m tall.  
How much taller is the LVAH?



The Kölntriangle is 103.2m tall.

LVA Hauptgebäude is 123m tall.

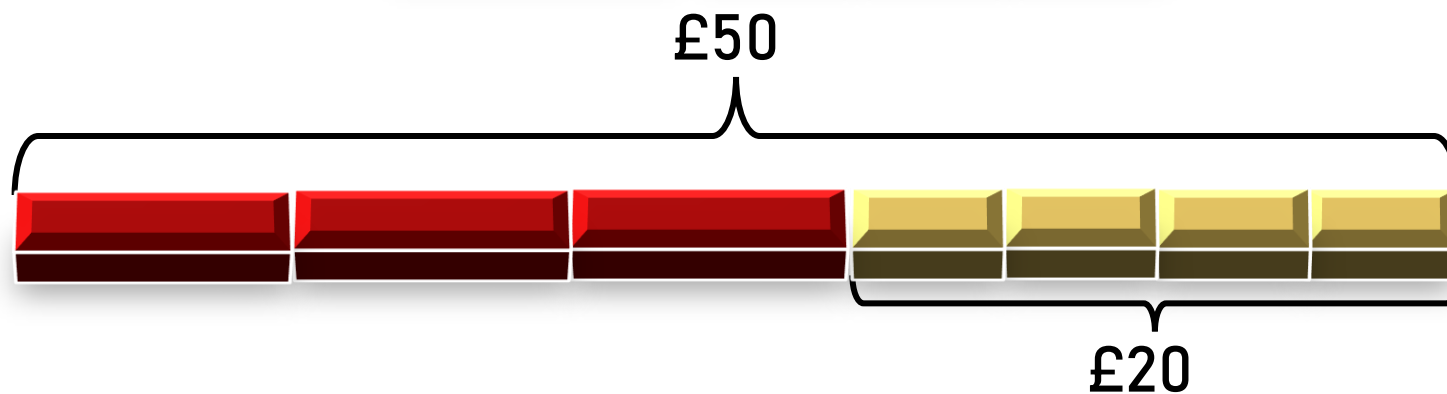
How much taller is the LVAH?

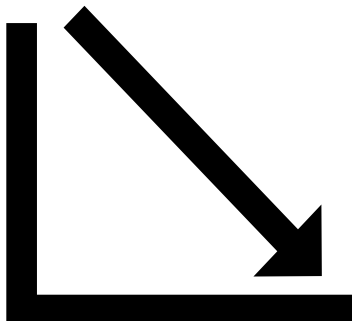


The Kölntriangle has 29 floors.  
LVA Hauptgebäude has 26 floors.  
How many more floors does the  
Kölntriangle have?

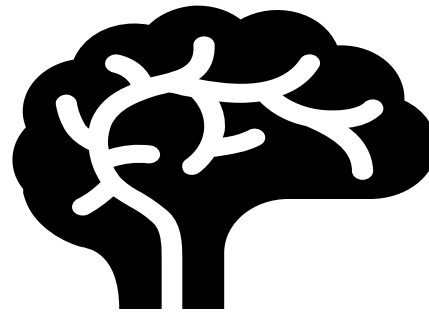


Siblings, Cindy, Kate and Charlie save £50 for a trip to the cinema.  
Cindy has £5 more than Kate. Charlie has £10 more than Cindy.  
How much does each child contribute to the kitty?

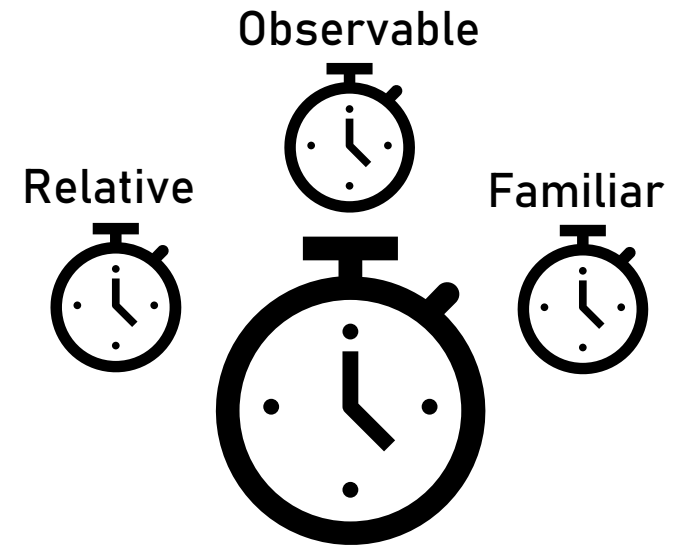




Demands on Attention



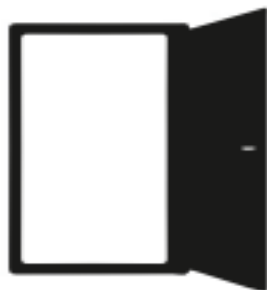
Mental Manipulation



Fluid



Identify the threshold concepts



Allow time- just like you should for yourself



Have multiple examples



Consider the context





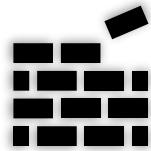
Integrative



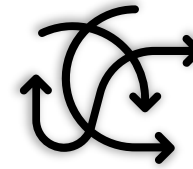
Transformative



Irreversible



Reconstitutive



Troublesome



Discursive

# Threshold Concepts

In "*Thinking Deeply about Primary Mathematics*" we explore threshold concepts in the primary mathematics classroom. With a hierarchical subject such as mathematics it is extremely difficult to pin down precise threshold concepts, which is why I was reticent to commit a list to print. Instead, what follows is a fluid list comprised of those concepts and ideas which our pupils will find particularly difficult to navigate and which will demand some serious consideration on our part.

- The principles of counting (stable order, 1:1 correspondence, cardinality, order irrelevance, abstract principle)
- Unitising
- Equality/equivalence
- Moving from cumbersome strategies to automatic recall of number facts
- The field axioms (laws of arithmetic)



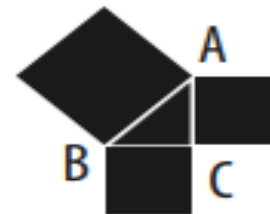
Be responsive in the moment



Keep the concept stable, alter the context



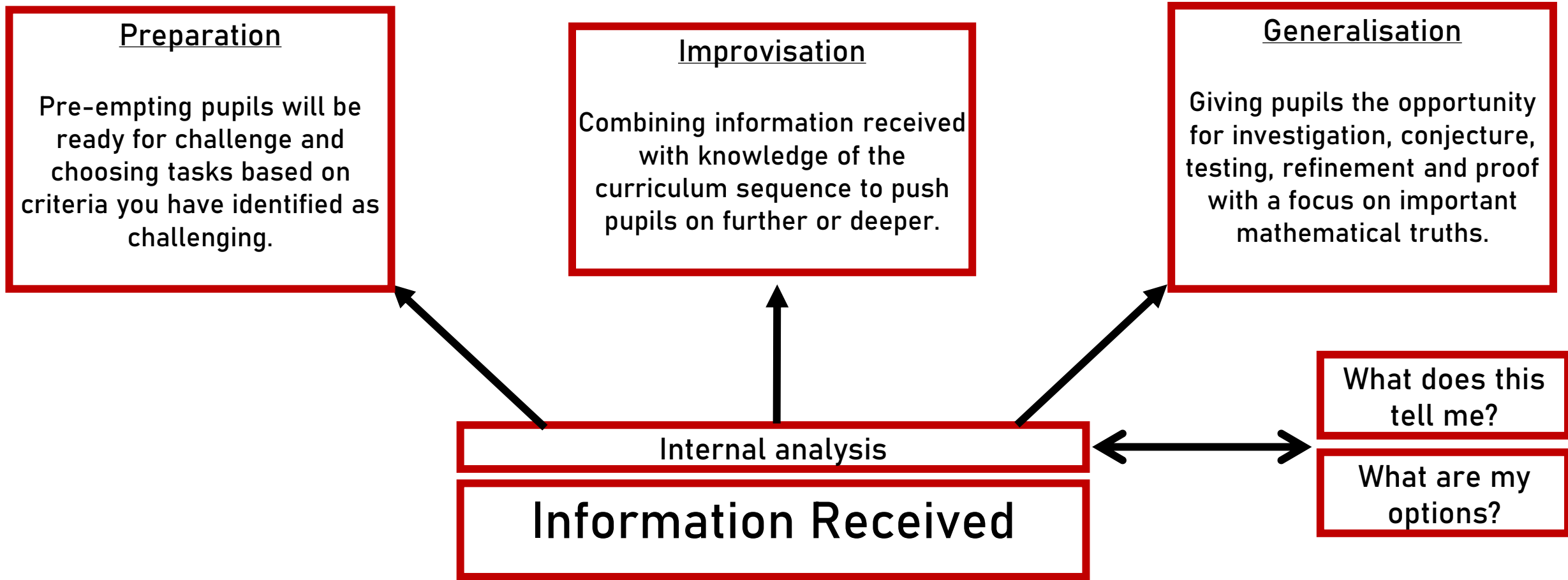
Draw on mathematically rich sources



Thinking like an exam writer







## What are my options? Generalisation

T  
A  
S  
K

Think of **three consecutive numbers**. These are your numbers.

Add your numbers.

Multiply your middle number by 3.

***What do you notice? Explain.***

## Making Mathematical Generalisations

### Identifying Patterns

$$1 + 2 + 3 = 6$$
$$2 + 3 + 4 = 9$$

“If I find the sum of  $3 + 4 + 5$  the total will be 12.”

### Making Conjectures

$$1 + 2 + 3 = 6$$
$$2 + 3 + 4 = 9$$
$$3 + 5 + 7 = 15$$

“If I find the sum of any 3 numbers the total will be a multiple of 3.”

## Providing Support for Mathematical Claims

### Providing Non-Proof Arguments

$$2 + 3 + 4 = 9$$

$$5 + 6 + 7 = 18$$

$$11 + 12 + 13 = 36$$

$$121 + 122 + 123 = 366 \quad \frac{366}{3} = 122$$

$$514 + 515 + 516 = 1545$$

$$\frac{1545}{3} = 515$$

“The sum of 3 consecutive numbers is a multiple of 3.”

### Providing Proofs

“Any 3 consecutive integers can be described as  $n$ ,  $n+1$  and  $n+2$ ”

$$2 + 3 + 4 = 9$$

$$11 + 12 + 13 = 36$$

“The additional units can be redistributed to make 3 equal groups.”



“Therefore, the sum of 3 consecutive integers is always a multiple of 3.”

E  
X  
P  
L  
A  
I  
N

**Agree or disagree:**

‘When I add two consecutive numbers, the answer is never in the two times tables.’

***Explain.***

## Making Mathematical Generalisations

### Identifying Patterns

### Making Conjectures

E  
X  
P  
L  
A  
I  
N

#### Agree or disagree:

'When I add two consecutive numbers, the answer is never in the two times tables.'

#### Explain.

## Providing Support for Mathematical Claims

### Providing Non-Proof Arguments

### Providing Proofs



Must convince the learner that the misconception is erroneous



Address the misconception head on

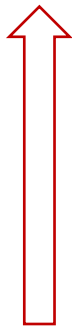
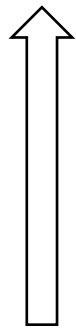
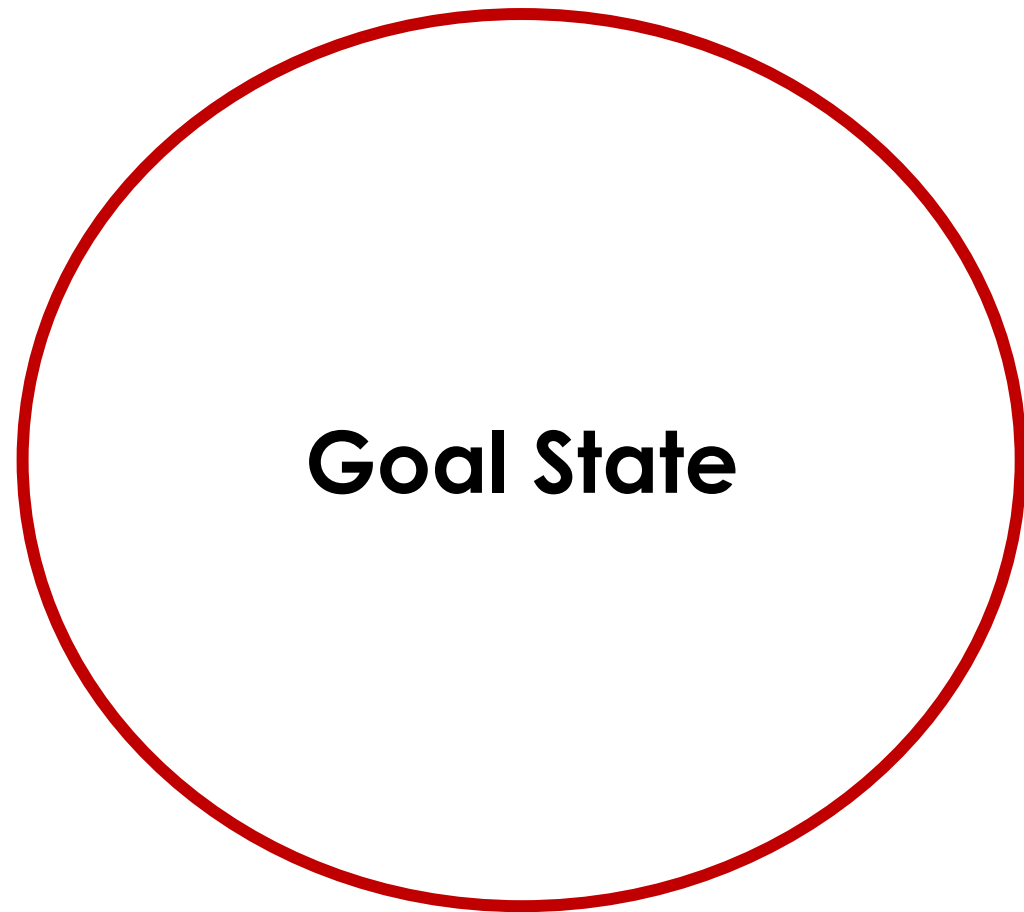
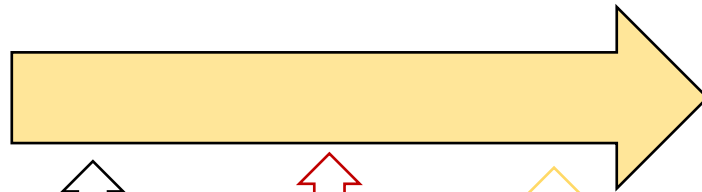
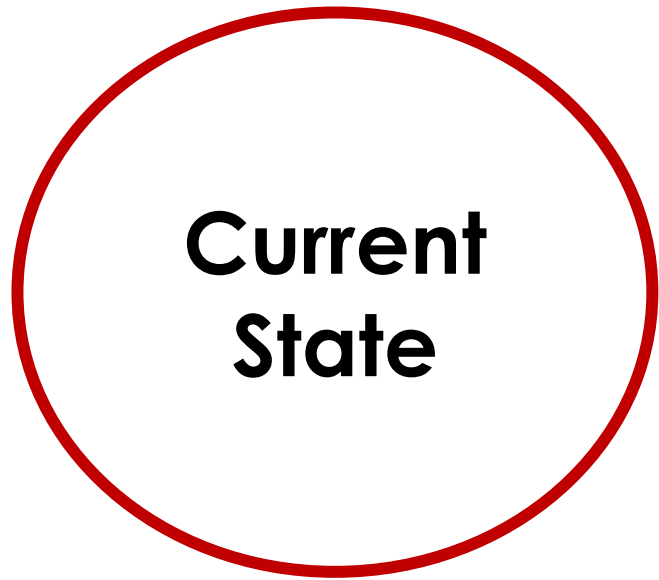


Misconceptions will never go away, we must build stronger, accurate schema over the top



Be aware of common misconceptions in advance





**A**

**B**

**C**

$$82 + 58$$

$$97 + 34$$

$$45 + 67$$

$$30 + 28$$

Rearrange these number sentences into formal written columns.



$$\begin{array}{r} 82 \\ + 58 \\ \hline \end{array}$$

$$\begin{array}{r} 97 \\ + 34 \\ \hline \end{array}$$

$$\begin{array}{r} 45 \\ + 67 \\ \hline \end{array}$$

$$\begin{array}{r} 69 \\ + 43 \\ \hline \end{array}$$

$$\begin{array}{r} 30 \\ + 28 \\ \hline \end{array}$$

$$\begin{array}{r} 30 \\ + 36 \\ \hline \end{array}$$

$$\begin{array}{r} 23 \\ + 70 \\ \hline \end{array}$$

$$\begin{array}{r} 33 \\ + 46 \\ \hline \end{array}$$

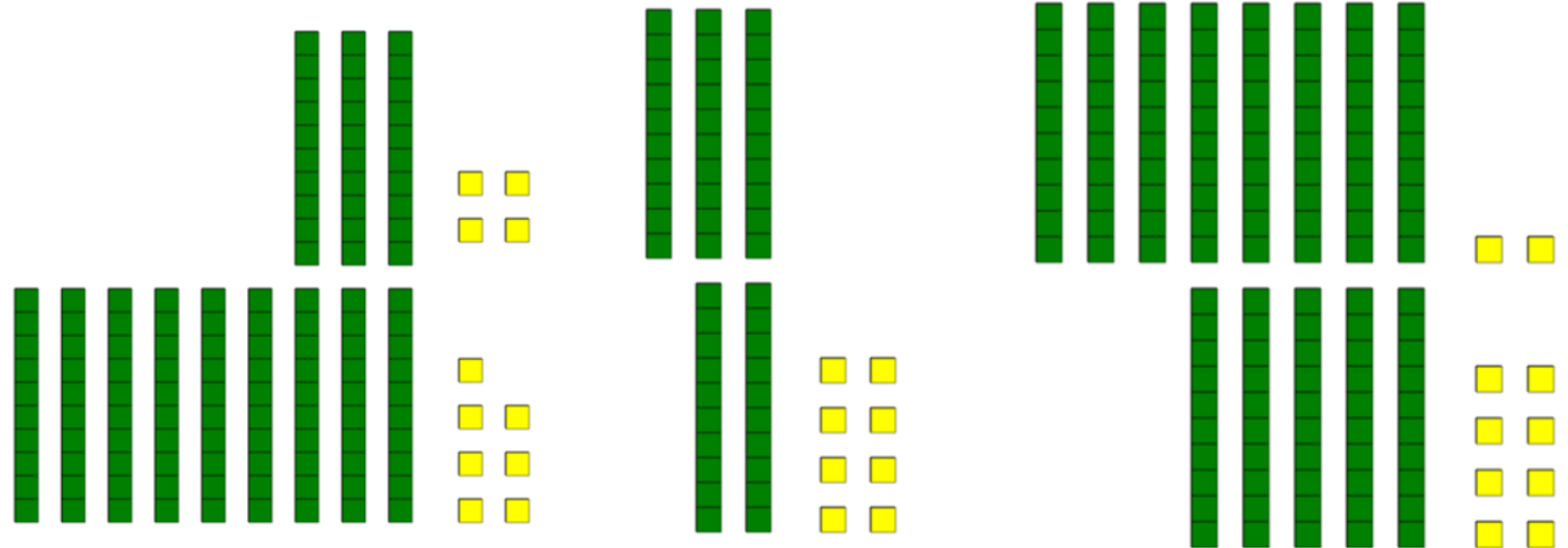
Tick the calculations which involve regrouping.

$$\begin{array}{r} 82 \\ + 58 \\ \hline \end{array}$$

$$\begin{array}{r} 97 \\ + 34 \\ \hline \end{array}$$

$$\begin{array}{r} 30 \\ + 28 \\ \hline \end{array}$$

$$\begin{array}{r} 30 \\ + 36 \\ \hline \end{array}$$



Match the calculations to their physical/pictorial representations

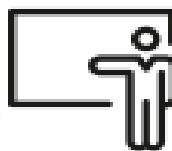
$$\begin{array}{r} 82 \\ + 58 \\ \hline \end{array}$$

$$\begin{array}{r} 97 \\ + 34 \\ \hline \end{array}$$

$$\begin{array}{r} 30 \\ + 28 \\ \hline \end{array}$$

$$\begin{array}{r} 30 \\ + 36 \\ \hline \end{array}$$

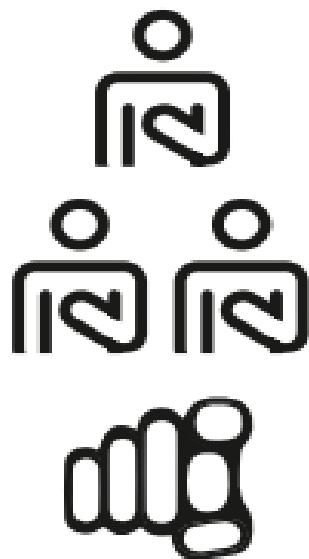
Calculate the sum of these pairs of numbers



Ask fewer questions initially



My turn, our turn, your turn



Choose a worked example

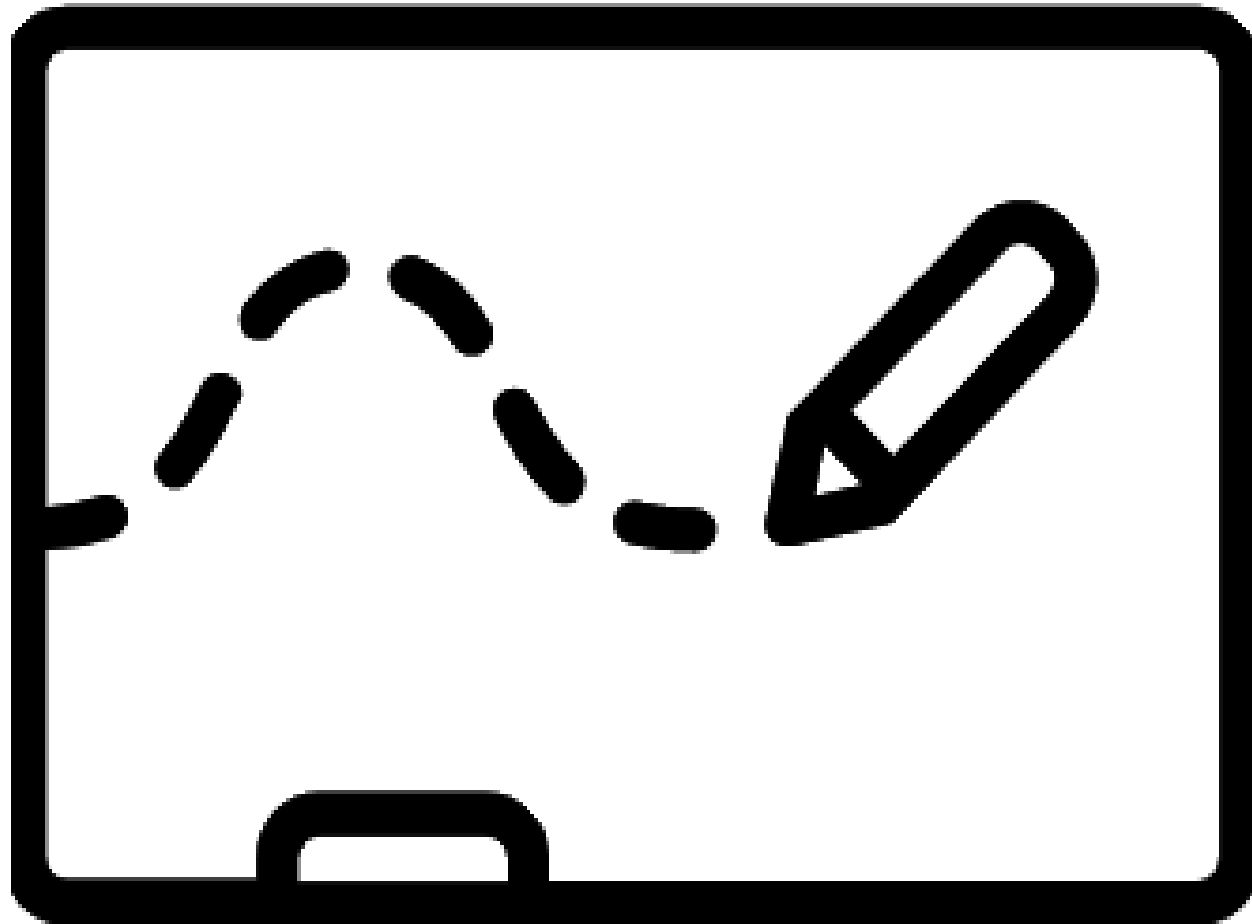


Use few words



My turn, our turn,  
your turn





# My Turn

$$342 - 20$$

$$\textcircled{1} 300 + 42$$

$$\textcircled{2} 42 - 20 = 22$$

$$\textcircled{3} 300 + 22 = 322$$

# My Turn

$$342 - 20$$

$$\textcircled{1} 300 + 42$$

$$\textcircled{2} 42 - 20 = 22$$

$$\textcircled{3} 300 + 22 = 322$$

# Our Turn

$$242 - 20$$



# Your Turn

$$442 - 20$$

# My Turn

$$342 - 20$$

# Your Turn

$$442 - 20$$

$$372 - 20$$



$$458 - 30$$



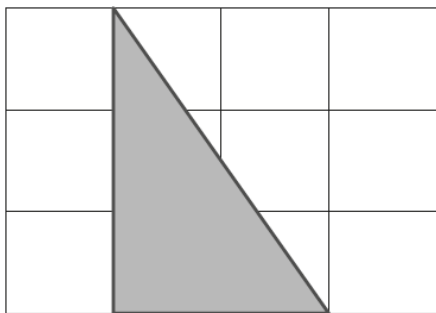
$$763 - 40$$

# Paired worked examples

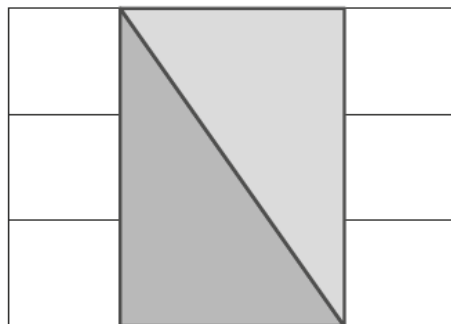
My Turn	Your Turn
<div data-bbox="453 674 1166 1125" style="border: 1px solid black; padding: 10px;"><math display="block">342 - 20</math><p>① <math>300 + 42</math></p><p>② <math>42 - 20 = 22</math></p><p>③ <math>300 + 22 = 322</math></p></div>	$240 - 20$

# Procedural worked example

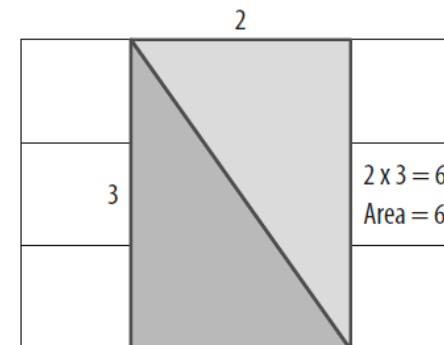
Pre-step one: (question selection)



Step one:

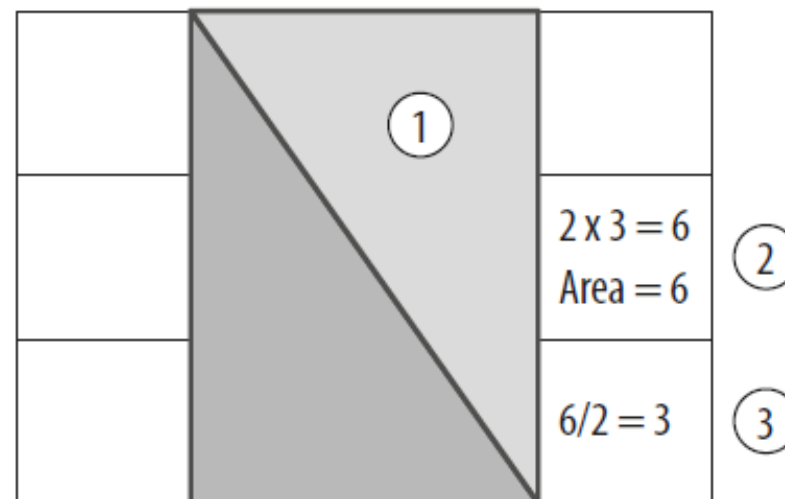
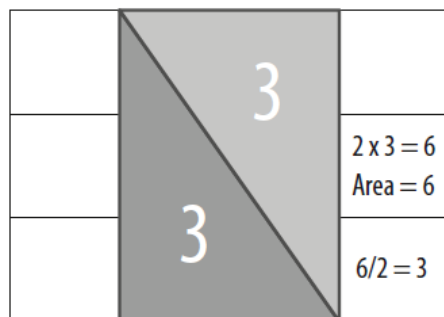


Step two:



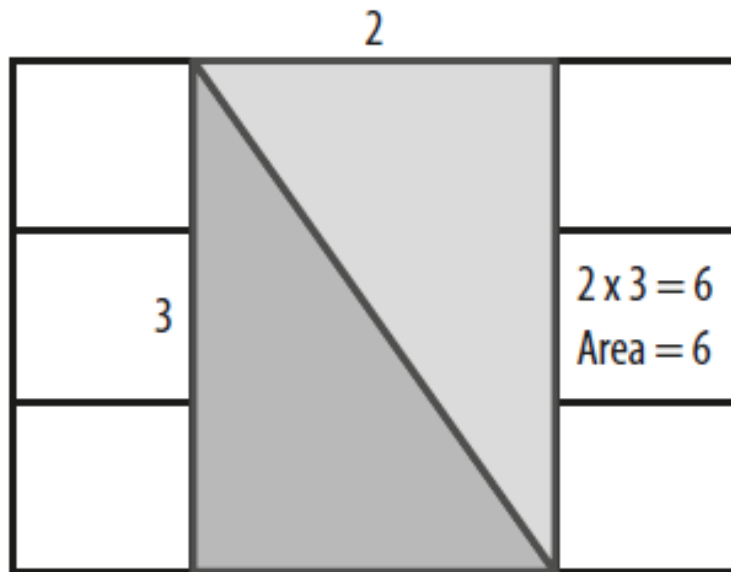
'Using what you know about the area of rectangles, find the area of this triangle.'

Step three:

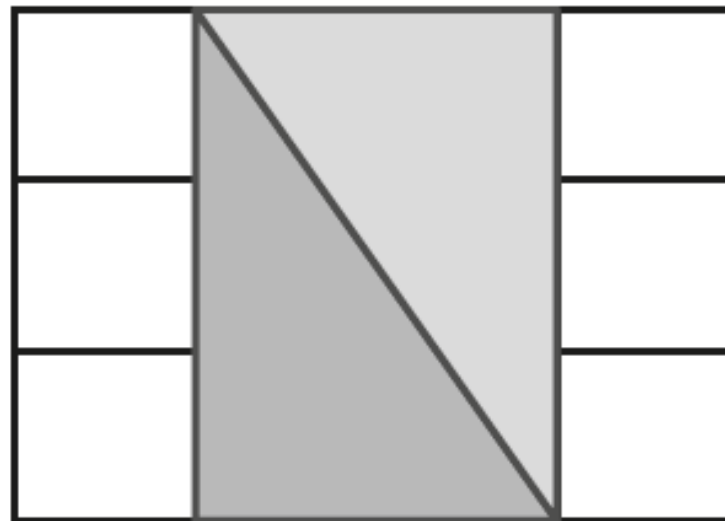


# Reverse phased worked example

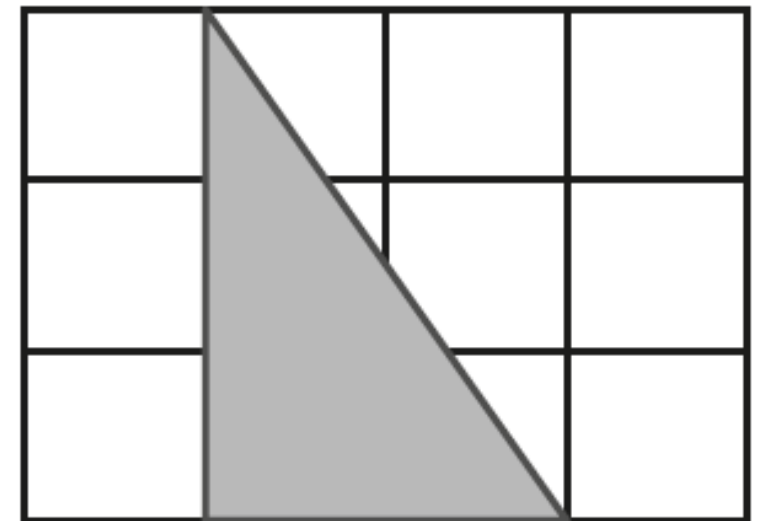
Calculate the area of the blue triangle.



Calculate the area of the blue triangle.



Calculate the area of the blue triangle.



# Reverse phased worked example

$$\begin{array}{r} \text{a.} \\ 325 \\ + 192 \\ \hline \phantom{3}17 \end{array}$$

$$\begin{array}{r} \text{b.} \\ 335 \\ + 192 \\ \hline \phantom{3}7 \end{array}$$

$$\begin{array}{r} \text{c.} \\ 325 \\ + 194 \\ \hline 9 \end{array}$$

$$\begin{array}{r} \text{d.} \\ 335 \\ + 194 \\ \hline \end{array}$$

# Problem worked examples

**Jack buys 6 packs of Euro 2021 football stickers. Each precious pack has 8 stickers with the images of famous football players. How many stickers does Jack buy altogether?**



# My Turn

**Jack buys 6 packs of Euro 2021 football stickers. Each precious pack has 8 stickers with the images of famous football players. How many stickers does Jack buy altogether?**

# Our Turn

**Jack buys 4 packs of Euro 2021 football stickers. Each precious pack has 8 stickers with the images of famous football players. How many stickers does Jack buy altogether?**



Make time to talk  
about language with  
experienced colleagues



Quality over quantity



Model the expected  
behaviours

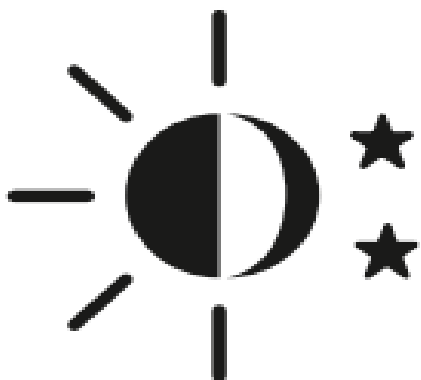


Build into the questions  
you ask over time

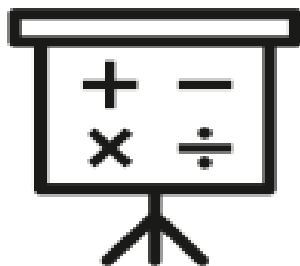




Consider reasoning a  
24/7 pursuit



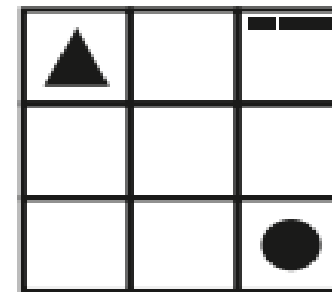
Plant expectations of  
reasoning firmly in  
mathematics

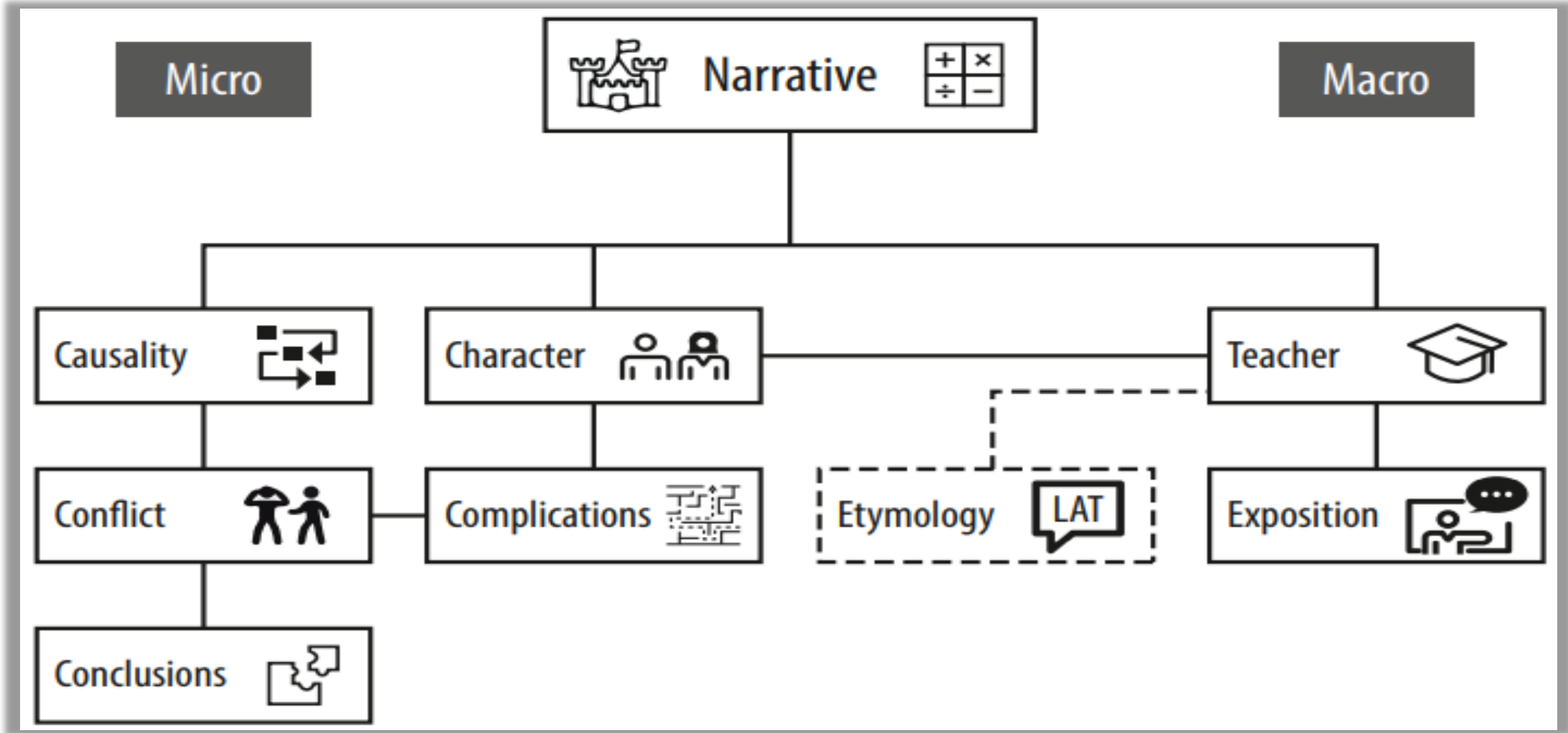


Provide scaffold for  
effective communication  
of ideas



Embrace recreational  
mathematics







Free yourself from  
imagined pressures



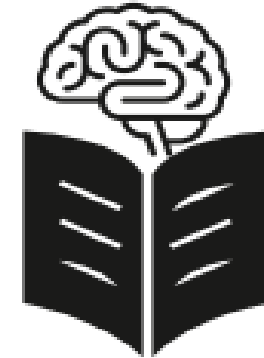
Begin with an end  
goal in mind



Draw on established  
mental models



Build in frequency  
and familiarity





Read little but often



Read things you don't agree with



Seek out the peer reviewed

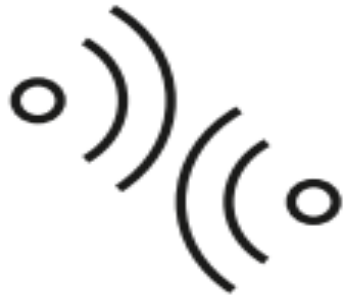


Revisit regularly





Embrace the echo



Block and mute at will

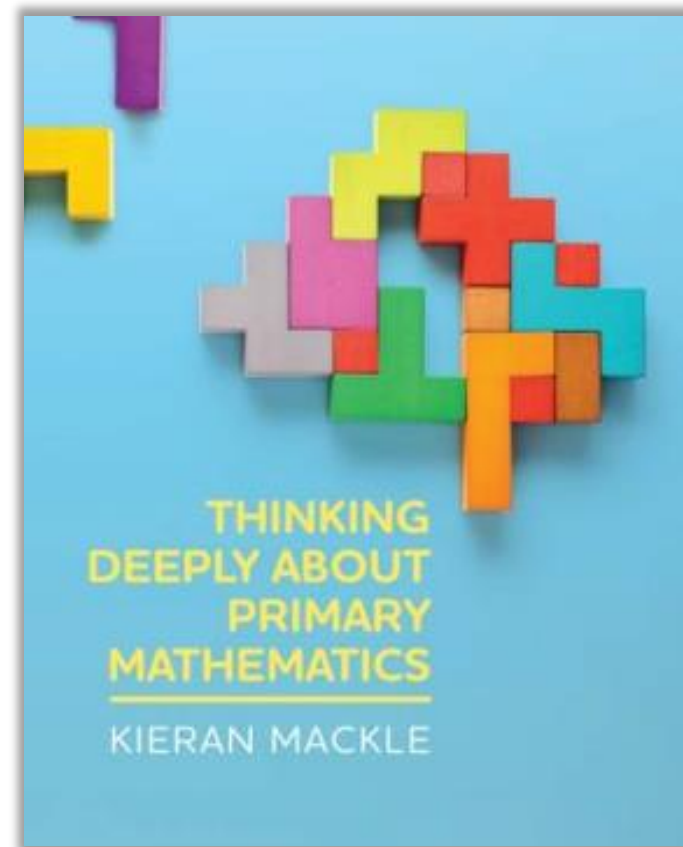
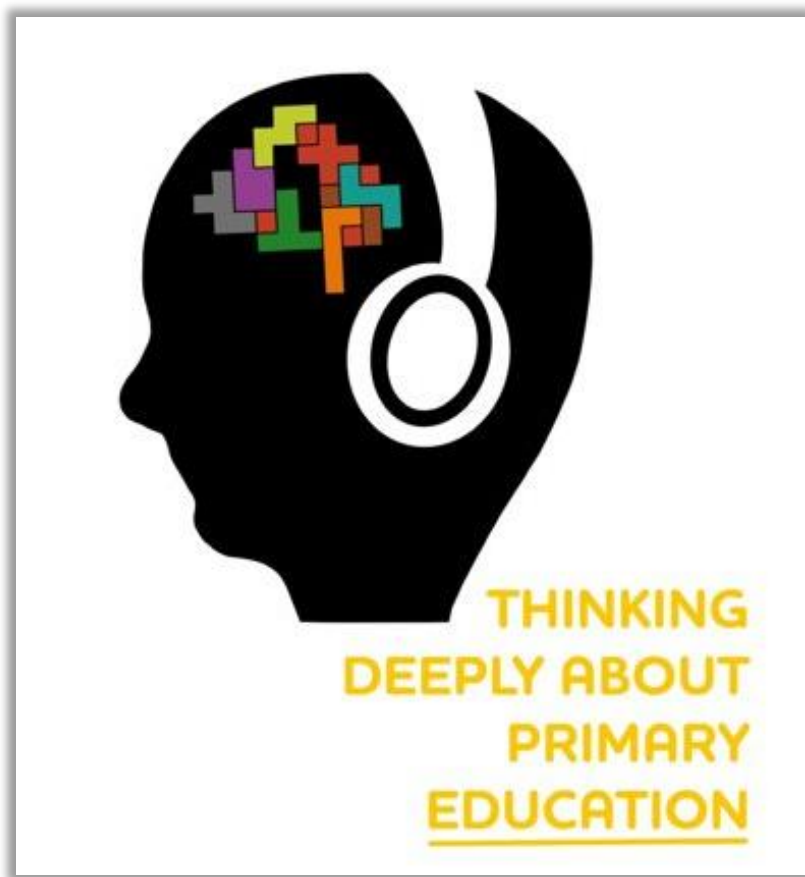


Follow the content  
not the person

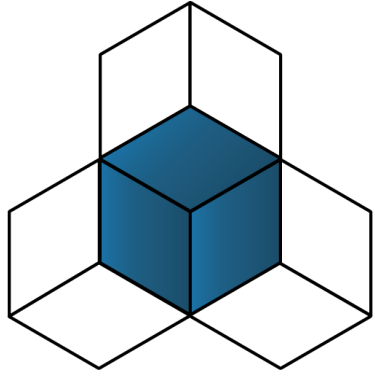


Switch off









# Complete Mathematics CPD

## Thinking Deeply about Primary Mathematics

[kieran.mackle@completemaths.com](mailto:kieran.mackle@completemaths.com)

@Kieran\_M\_Ed