# Embedding Elements of TfM in post-16 Maths

*Vicky Lally & Adam Creen*

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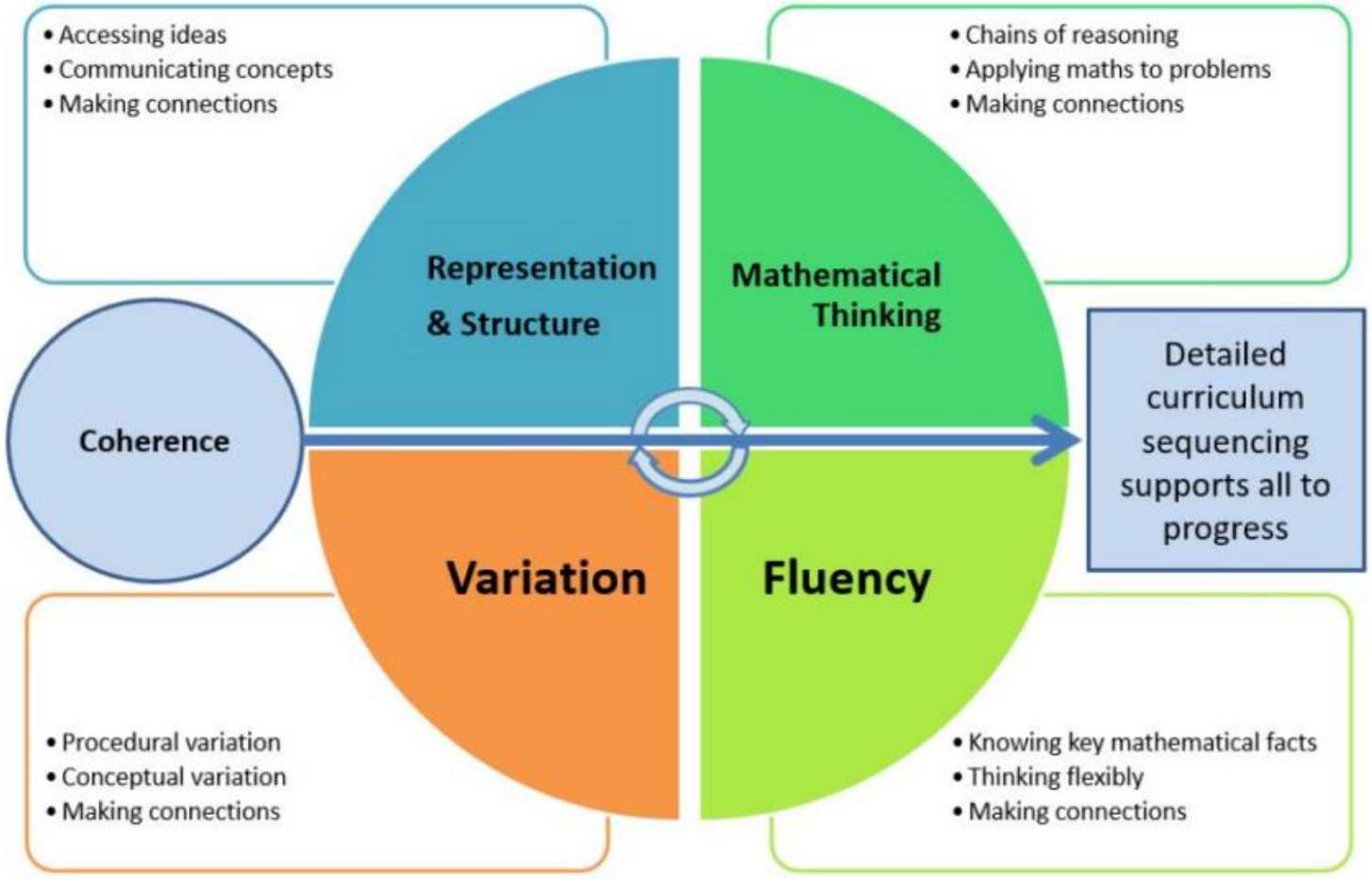
# Aims

- Through discussion, share simple ideas that bring the brief of Teaching for Mastery into the post-16 environment
- Use a variety of contexts to show these ideas are applicable to both Core Maths & A Level Maths, whether your school actively embraces TfM or not

- Vicky and Adam are A Level teachers at SJB and Salesian respectively

# The 5 Big Ideas

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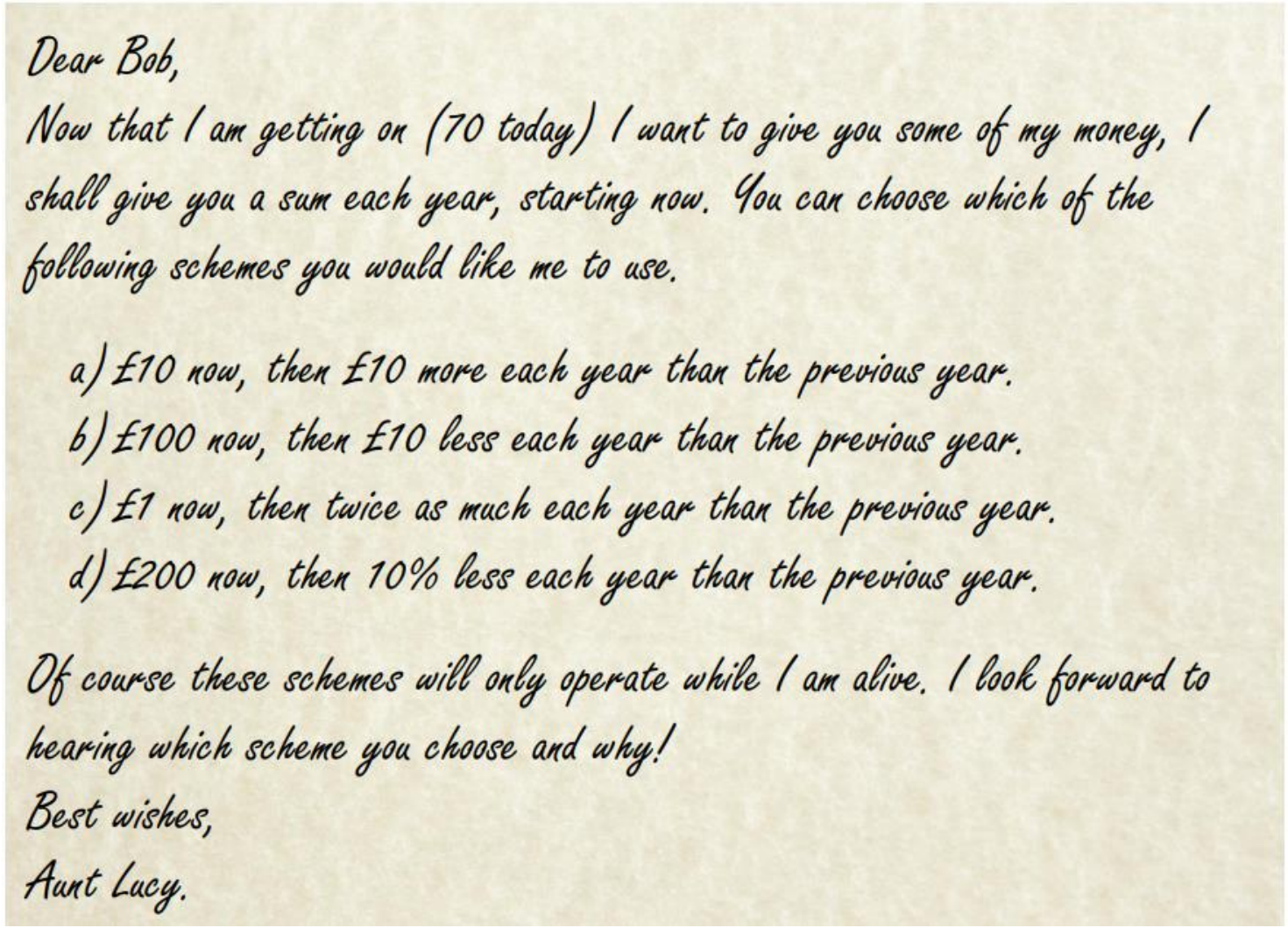
# Sequences (through the eyes of Core Maths)

Note: we have delivered the Edexcel “Maths in Context” Level 3 course, where – along with all Level 3 courses – the emphasis is on the in “context”

To sow an initial seed of “thinking” a task can enable students to actively think about two standard sequences in mathematics (the arithmetic and geometric sequence) without mentioning them explicitly.

# The Stimulus

- Note this task I first met in 1993
- It has been adapted to reach as many parts of the specification as possible (APs, GPs, percentages, spreadsheets)
- Importantly, there is a real-life application to engage students



Dear Bob,

Now that I am getting on (70 today) I want to give you some of my money, I shall give you a sum each year, starting now. You can choose which of the following schemes you would like me to use.

a) £10 now, then £10 more each year than the previous year.

b) £100 now, then £10 less each year than the previous year.

c) £1 now, then twice as much each year than the previous year.

d) £200 now, then 10% less each year than the previous year.

Of course these schemes will only operate while I am alive. I look forward to hearing which scheme you choose and why!

Best wishes,

Aunt Lucy.

# What discussion can we get out of this:

| | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Year | Age 70 | Scheme A | Total A | Scheme B | Total B | Scheme C | Total C | Scheme 4 | Total 4 |
| 2 | 1 | 70 | 10 | 10 | 100 | 100 | 1 | 1 | 200 | 200 |
| 3 | 2 | 71 | 20 | 30 | 90 | 190 | 2 | 3 | 180 | 380 |
| 4 | 3 | 72 | 30 | 60 | 80 | 270 | 4 | 7 | 162 | 542 |
| 5 | 4 | 73 | 40 | 100 | 70 | 340 | 8 | 15 | 145.8 | 687.8 |
| 6 | 5 | 74 | 50 | 150 | 60 | 400 | 16 | 31 | 131.22 | 819.02 |
| 7 | 6 | 75 | 60 | 210 | 50 | 450 | 32 | 63 | 118.098 | 937.118 |
| 8 | 7 | 76 | 70 | 280 | 40 | 490 | 64 | 127 | 106.2882 | 1043.4062 |
| 9 | 8 | 77 | 80 | 360 | 30 | 520 | 128 | 255 | 95.65938 | 1139.06558 |
| 10 | 9 | 78 | 90 | 450 | 20 | 540 | 256 | 511 | 86.093442 | 1225.15902 |
| 11 | 10 | 79 | 100 | 550 | 10 | 550 | 512 | 1023 | 77.4840978 | 1302.64312 |
| 12 | 11 | 80 | 110 | 660 | 0 | 550 | 1024 | 2047 | 69.735688 | 1372.37881 |
| 13 | 12 | 81 | 120 | 780 | 0 | 550 | 2048 | 4095 | 62.7621192 | 1435.14093 |
| 14 | 13 | 82 | 130 | 910 | 0 | 550 | 4096 | 8191 | 56.4859073 | 1491.62683 |
| 15 | 14 | 83 | 140 | 1050 | 0 | 550 | 8192 | 16383 | 50.8373166 | 1542.46415 |
| 16 | 15 | 84 | 150 | 1200 | 0 | 550 | 16384 | 32767 | 45.7535849 | 1588.21774 |
| 17 | 16 | 85 | 160 | 1360 | 0 | 550 | 32768 | 65535 | 41.1782264 | 1629.39596 |
| 18 | 17 | 86 | 170 | 1530 | 0 | 550 | 65536 | 131071 | 37.0604038 | 1666.45637 |
| 19 | 18 | 87 | 180 | 1710 | 0 | 550 | 131072 | 262143 | 33.3543634 | 1699.81073 |
| 20 | 19 | 88 | 190 | 1900 | 0 | 550 | 262144 | 524287 | 30.0189271 | 1729.82966 |
| 21 | 20 | 89 | 200 | 2100 | 0 | 550 | 524288 | 1048575 | 27.0170344 | 1756.84669 |
| 22 | 21 | 90 | 210 | 2310 | 0 | 550 | 1048576 | 2097151 | 24.3153309 | 1781.16202 |
| 23 | 22 | 91 | 220 | 2530 | 0 | 550 | 2097152 | 4194303 | 21.8837978 | 1803.04582 |
| 24 | 23 | 92 | 230 | 2760 | 0 | 550 | 4194304 | 8388607 | 19.695418 | 1822.74124 |
| 25 | 24 | 93 | 240 | 3000 | 0 | 550 | 8388608 | 16777215 | 17.7258762 | 1840.46711 |
| 26 | 25 | 94 | 250 | 3250 | 0 | 550 | 16777216 | 33554431 | 15.9532886 | 1856.4204 |
| 27 | 26 | 95 | 260 | 3510 | 0 | 550 | 33554432 | 67108863 | 14.3579598 | 1870.77836 |
| 28 | 27 | 96 | 270 | 3780 | 0 | 550 | 67108864 | 134217727 | 12.9221638 | 1883.70053 |
| 29 | 28 | 97 | 280 | 4060 | 0 | 550 | 134217728 | 268435455 | 11.6299474 | 1895.33047 |
| 30 | 29 | 98 | 290 | 4350 | 0 | 550 | 268435456 | 536870911 | 10.4669527 | 1905.79743 |
| 31 | 30 | 99 | 300 | 4650 | 0 | 550 | 536870912 | 1073741823 | 9.42025739 | 1915.21768 |
| 32 | 31 | 100 | 310 | 4960 | 0 | 550 | 1073741824 | 2147483647 | 8.47823166 | 1923.69592 |

- Formulae in spreadsheets
- Increasing and Decreasing Arithmetic Sequences
- Increasing and Decreasing Geometric Sequences
- Summation of a Sequence
- Percentage Reduction
- Recurrence Relationship
- A question with “no right answer” – how did students deal with this?
- (Link with the King’s Chessboard)

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# What about A Level Maths?

- Tempting to just teach “the formulas”
- Two for students to learn, two given in the formula booklet

- What’s at the heart of the formula?

$$n - 1$$

- Harry the Hare
- We need them to have “structural understanding”

# Justifying the summation

- writing the sum from 1 to 100
- reversing the sum

- adding pairs of terms and seeing they are all 101

- How many pairs? But this is double the total
- So we get $\frac{100}{2}(101) = 5050$


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# Linking topics in A Level

- Sigma notation
- But also … how many terms from 10 to 20?

$$\sum_{10}^{20} r$$

- Is this the same problem as Binomial inequalities?

$$P(10 \leq X \leq 20)$$

# To summarise

- At A Level we want

  structural understanding

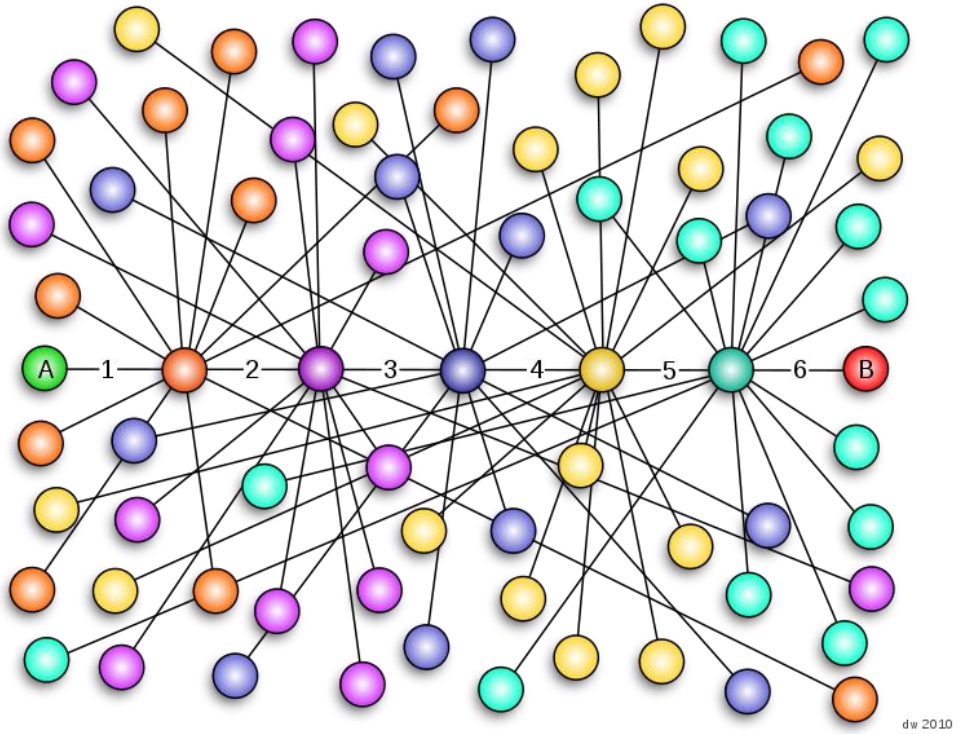
  the $n^{th}$ term to be “obvious”

  AND learning the formulas and proofs (of course!)

# Standard Deviation

- This is not always the most intuitive idea for a student, so whether a Core Maths mathematician or A-level mathematician, it is important students get a feel as to:
    - why we need measure of dispersion
    - the “feel” of dispersion (which then supports Range & IQR)
    - where the formula originated from!

# The Stimulus




**Six degrees of separation** is the theory that everyone is six or fewer steps away, by way of introduction, from any other person in the world, so that a chain of "a friend of a friend" statements can be made to connect any two people in a maximum of six steps.

In 1993 Will Smith starred in a film “Six degrees of separation”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# Students appreciating dispersion

A new student is going to join Mrs Skelton's psychology class and our maths class. Each student in the two classes finds out how many connections there are between themselves and the new student:

| Class 1 | |
|---|---|
| **Student** | **Number of connections, $x$** |
| Katrina | 1 |
| Abdul | 1 |
| Erica | 3 |
| Heidi | 1 |
| Maxine | 2 |
| Reatha | 4 |
| Garrett | 3 |
| Diane | 8 |
| Jacinda | 1 |
| Jessica | 2 |
| Nickolas | 3 |
| Christine | 10 |
| Renee | 2 |
| Yasuko | 3 |
| Justin | 6 |
| Marguerite | 1 |

| Class 2 | |
|---|---|
| **Student** | **Number of connections, $x$** |
| Jean | 6 |
| Paula | 4 |
| Francis | 2 |
| Gustavo | 4 |
| Laurence | 3 |
| Naomi | 2 |
| Richard | 2 |
| Alpita | 5 |
| Christopher | 3 |
| Sarah | 4 |
| Tom | 2 |
| Tyrone | 3 |
| Claire | 4 |
| Laurence | 2 |
| Penny | 3 |
| Malena | 2 |

1: This means they know the person
2: This means one of their friends knows the person

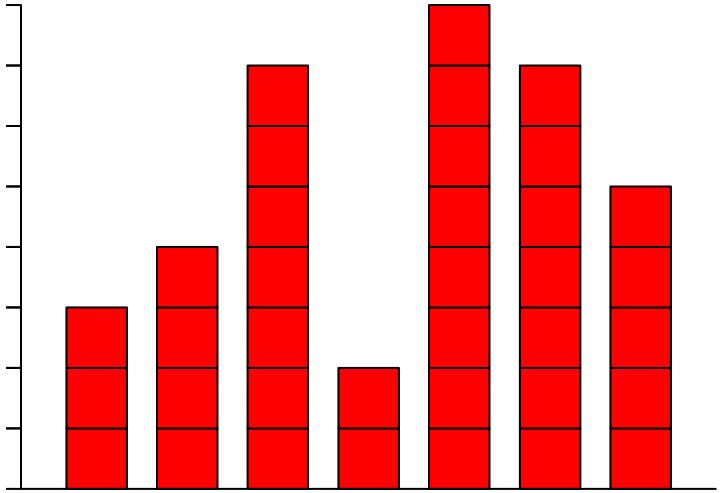
Find the mean number of connections this student has with Class 1 and Class 2, and write a conclusion.

# Does the student have a “feel” for the mean?

Here is a set of data values: 3 4 6 2 8 7 5

Find the mean

Does it feel right?



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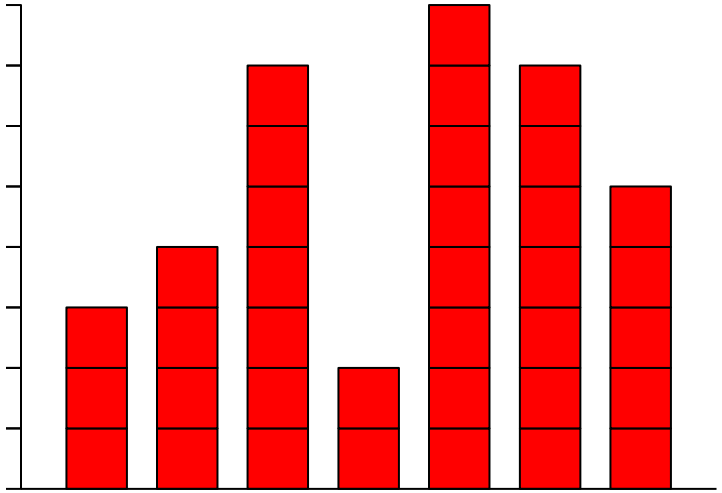

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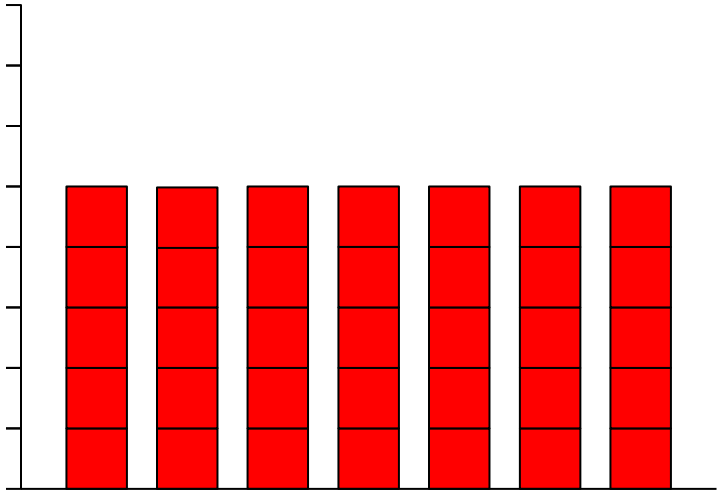


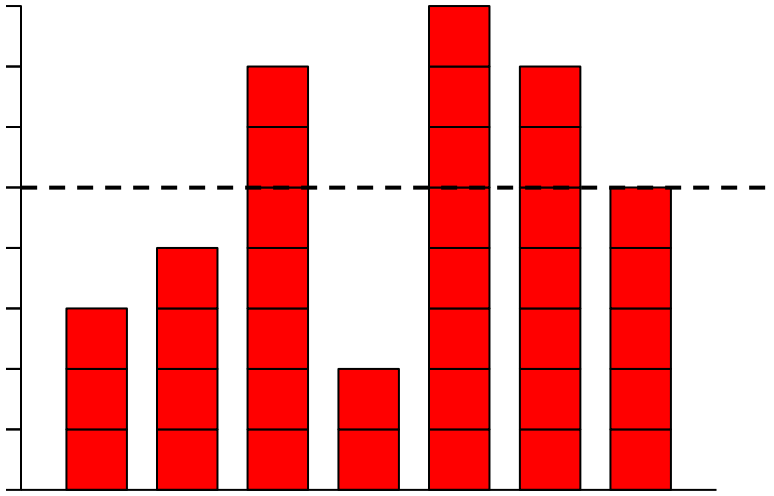
This can then lead to the development of the formulae for standard deviation

# Does the student have a “feel” for the mean?

Here is a set of data values: 3 4 6 2 8 7 5

Find the mean

Does it feel right?



# Using Desmos to “feel” dispersion

Desmos is great for statistics.

A dotplot is a simple graph where students can “see” each individual data value

To enter a list of data, simply:

| | |
|---|---|
| L = [enter data separated by columns] | note: square brackets |
| dotplot(L) | note: round brackets |

Students could tackle coding, even though not part of this course



# Introducing Standard Deviation at A Level

3 sets of data

1, 1, 3, 3, 5, 5, 7, 7

1, 2, 2, 4, 4, 6, 6, 7

1, 2, 2, 2, 6, 6, 6, 7

What are their means? Their medians?
Their ranges? Their inter-quartile ranges?

# 3 sets of data

1 1 *2* 3 3 *4* 5 5 *6* 7 7

1 2 *2* 2 4 *4* 4 6 *6* 6 7

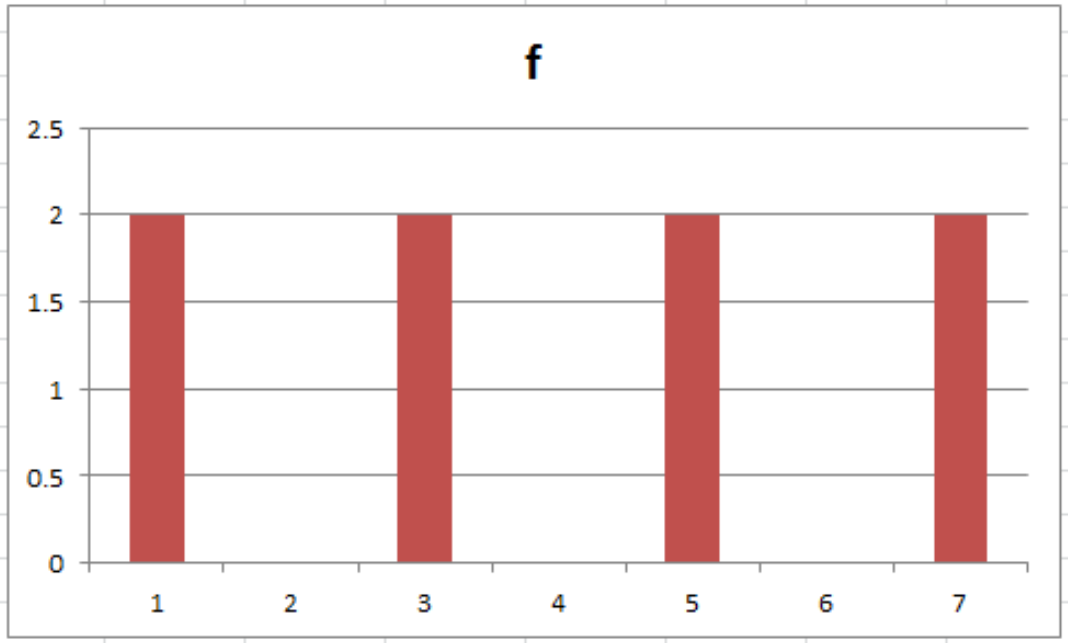
1 2 *2* 2 2 *4* 6 6 *6* 6 7

means and medians all 4

ranges all 6

inter-quartile ranges all 4

so how do we distinguish them?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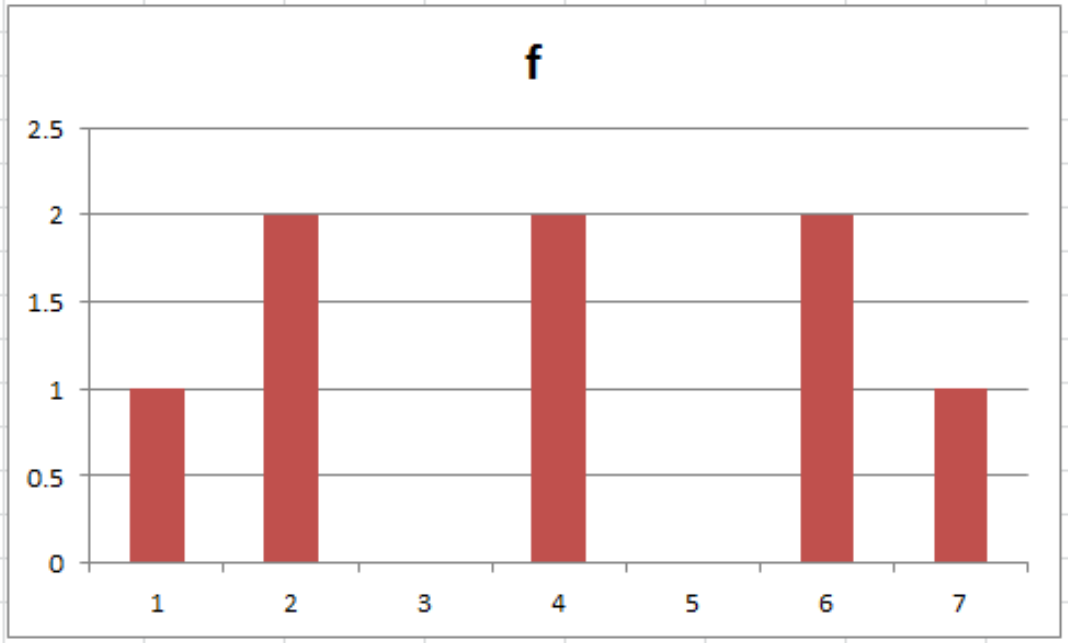
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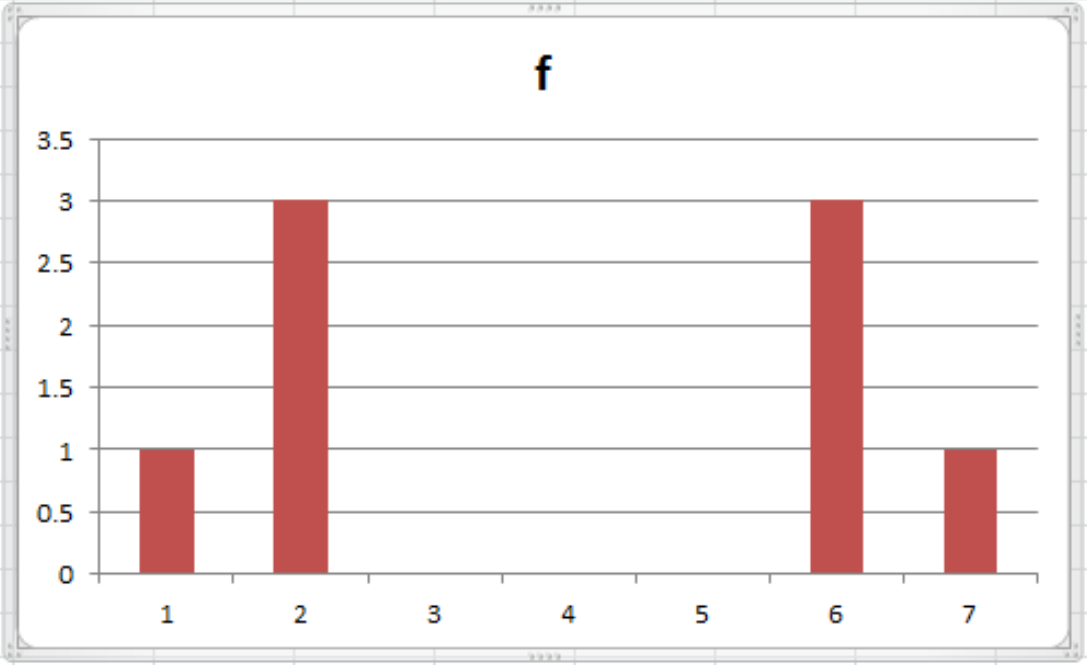
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0
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6
7

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# Inventing a measure of spread

- We could add up all the positive and negative distances from the mean
  - but this would add up to zero
- We could add up the magnitudes/moduli of the distances from the mean
  - and this would work, but not really reflect the effect of the furthest distances
- **We add up the squares of the distances from the mean and then average them**

standard deviation = the spread of the data using every data value measured in the same units as the data

$$\sigma = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2}$$

MSMSM = the mean of the squares

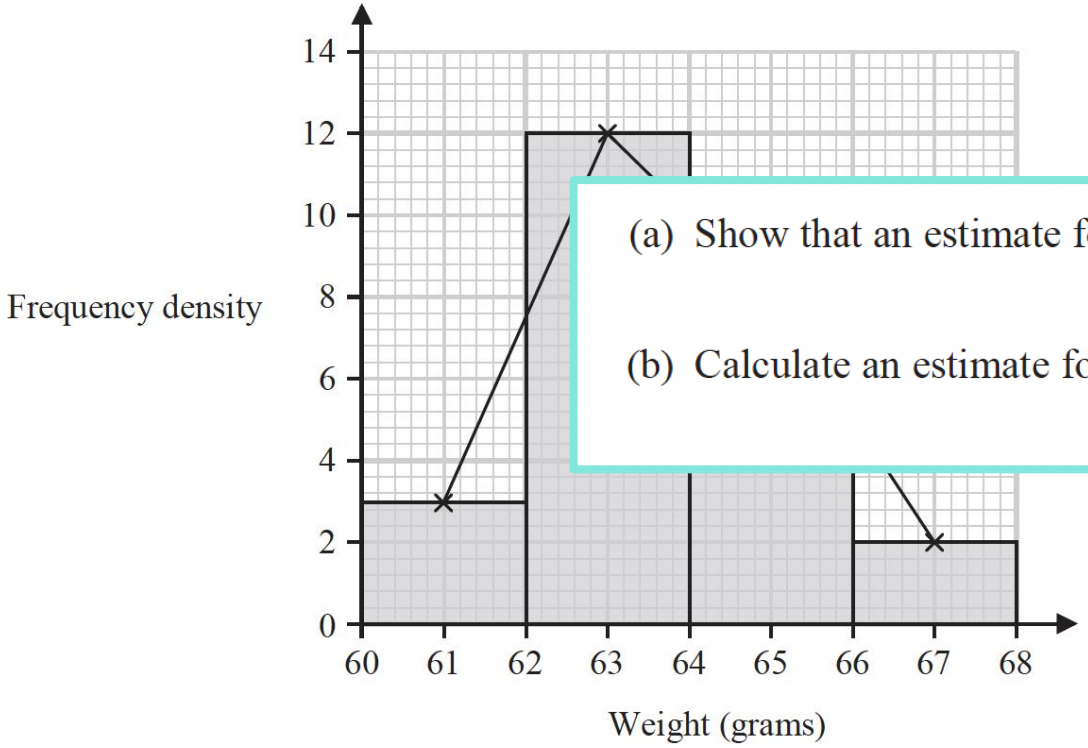
minus the square of the mean

# Teaching for understanding

- Giving students a formula is not enough
- How do we ensure they understand the concept?

**1.** The histogram and its frequency polygon below give information about the weights, in grams, of 50 plums.

Edexcel AS Maths June 2023




(a) Show that an estimate for the mean weight of the 50 plums is 63.72 grams.

**(2)**

(b) Calculate an estimate for the standard deviation of the 50 plums.

**(2)**

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# Teaching for understanding

Later it was discovered that the scales used to weigh the plums were broken.

Each plum actually weighs 5 grams less than originally thought.

(c) State the effect this will have on the estimate of the standard deviation in part (b).
Give a reason for your answer.

**(1)**

# Variation – students making predictions

- What if we increase all the values by 5? (link to coding)
- What if we double all the values?
- What if we do both?

- What if we remove the highest value (an outlier)?
- What if we add another value equal to the lowest?

# Variation – more interesting

- What if we add a value equal to the mean?
- How big does an outlier have to be to have an effect?

# Variation in using the formula

- Data set
- Summary statistics
- Summary statistics and an additional value
  so they think what changes, what has stayed the same
- Given standard deviation, mean, number of values, can they find $\sum x^2$ ?

# Variation in using the formula

- Given some working, can they spot the error? Can you?

| Shoe Size, x | Frequency, f | | |
|---|---|---|---|
| 5 | 7 | $7 \times 5 = 35$ | $7^2 \times 5 = 245$ |
| 6 | 9 | $9 \times 6 = 54$ | $9^2 \times 6 = 486$ |
| 7 | 10 | $10 \times 7 = 70$ | $10^2 \times 7 = 700$ |
| 8 | 4 | $4 \times 8 = 32$ | $4^2 \times 8 = 128$ |
| | $n = 30$ | $\Sigma x = 191$ | $\Sigma x^2 = 1559$ |

- All these force thinking and discussion, as opposed to repetitive calculation

# And now some updates…

- The AMSP is a government-funded initiative, managed by MEI, providing national support for teachers and students in all state-funded schools and colleges in England.
- It aims to increase participation in AS/A level Mathematics and Further Mathematics, and Core Maths, and improve the teaching of these qualifications.
- Additional support is given to those in priority areas to boost social mobility so that, whatever their gender, background or location, students can choose their best maths pathway post-16, and have access to high quality maths teaching.

# AMSP Local Area Coordinator



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AdMathsSouth


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# Summer Term Updates

- AMSP Transition Courses for Students
- SUMS online magazine for Year 12 and Year 13
- Extended PD Courses - now open for enrolment
- Subject Knowledge Live and KS5 Coordinator - open for enrolment soon
- On Demand Professional Development (ODPD)
- Sign up to Stay Informed

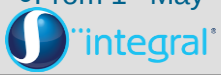
# AMSP Transition Courses

- **Essential Skills materials**
- Freely available on the AMSP website

- **Core Maths Skills course**
- Teachers can sign up their students
- OR
- Students can sign up independently
- From 1st May


- **Transition to A level Maths course**
- Teachers can sign up their students
- OR
- Students can sign up independently
- From 1st May


- **Transition to University course**
- Students sign up independently
- From 1st May




**https://amsp.org.uk/resource/transition**

# Steps to University Maths (SUMS):

- **SUMS - a new web-magazine for your Year 12 and Year 13 students**
- **Online magazine aimed at Y12 and Y13**
- **Different sections including:**
  - **Where can maths take you?** - Interviews with maths graduates.
  - **Preparing for university** – advice on what you should/could be doing now.
  - **Studying at university** – the lowdown from students and lecturers about what it's like to study maths.
  - **Do some maths** – puzzles, problems and new maths topics, plus the chance to get your work published in future issues.
- New issues are published on the second Monday of every month – hosted on Integral.
- https://amsp.org.uk/sums-steps-to-university-for-mathematical-students/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# Extended PD Courses

*Are you teaching something new?*

*Do you have a colleague who needs in-depth training?*

**AMSP extended PD courses** offer comprehensive support over several months and include access to dedicated course resources.

- Teaching A level Mathematics (TAM)
- Teaching Further Mathematics (TFM1 & TFM2)
- Teaching Mechanics (TM1 & TM2)
- Teaching Statistics (TS1 & TS2)
- Teaching Discrete Mathematics (TD)

amsp.org.uk/events/extended-pd-courses

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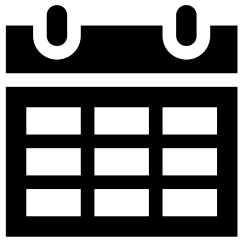
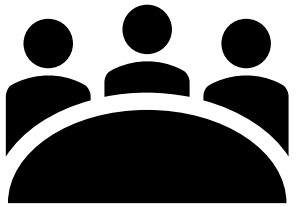
# Extended courses: comprehensive support

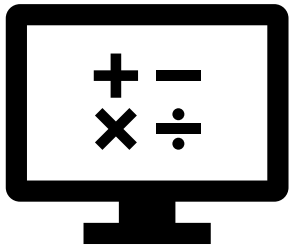


- Continuous support of up to two years

- Dedicated course tutors

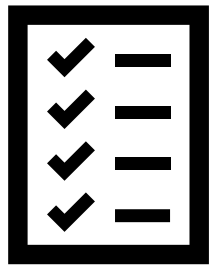


- Peer support



- Online sessions and/or study days

- Course-specific teaching resources



- Assignment with feedback

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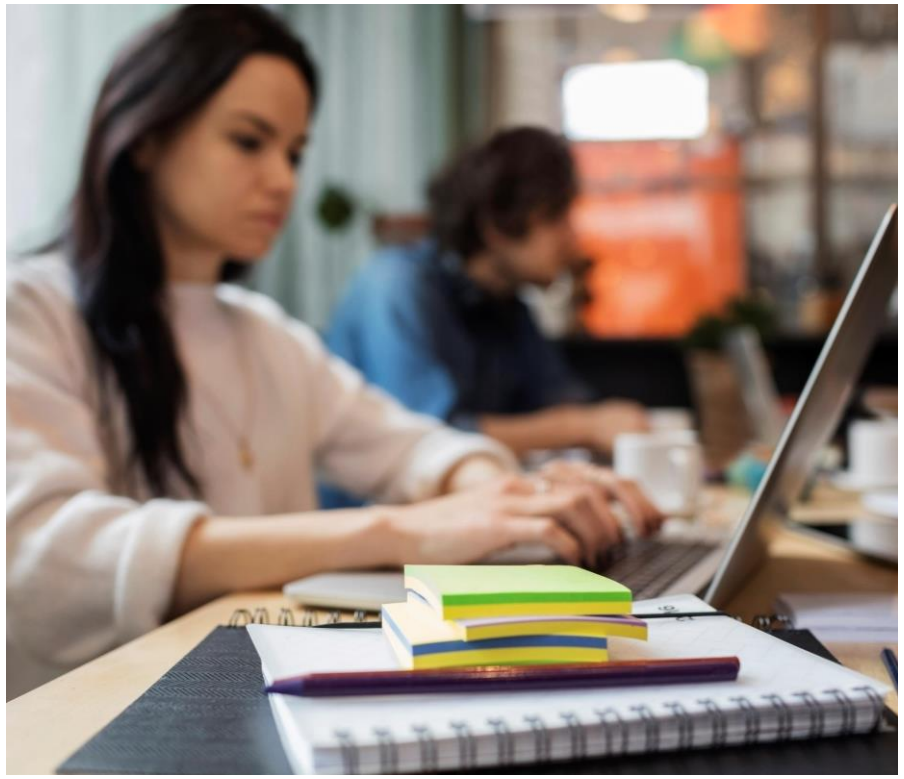
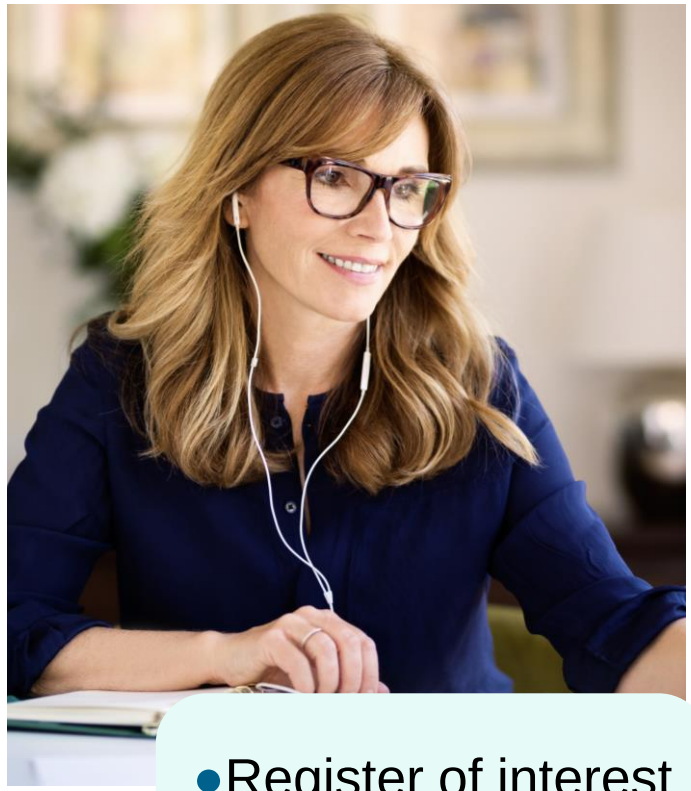
# Subject Knowledge Live and KS5Co



**Overview**

- One term, weekly sessions, 90 minutes each.
- Synchronous sessions modelling subject knowledge delivery including use of technology
- A year of access to course-specific teacher resources and recordings of the live sessions
- Free to teachers in state-funded schools in England

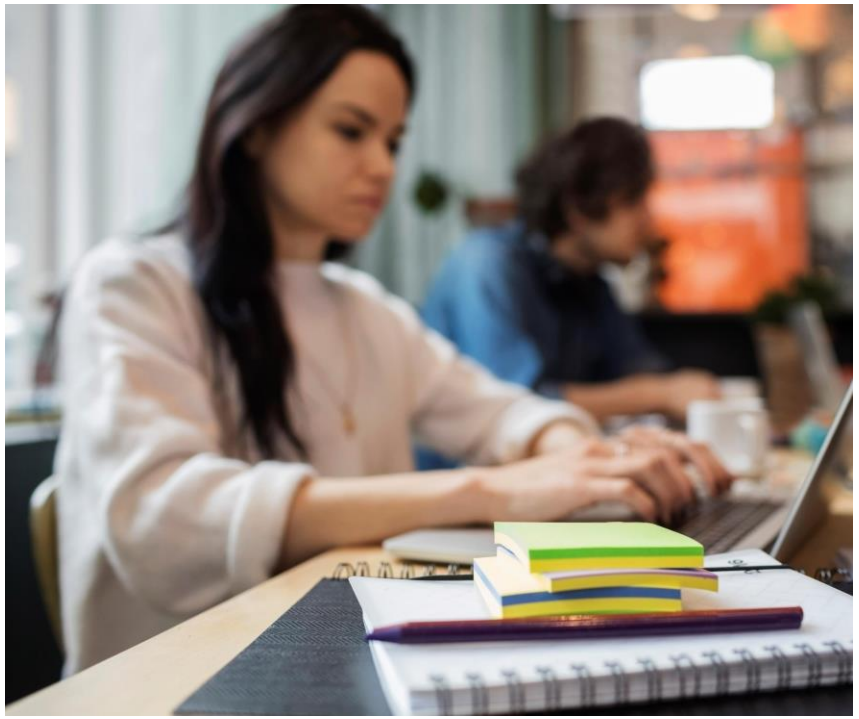
# Available courses



- Register of interest if nothing currently available

- Higher GCSE topics through mathematical thinking
- Core Maths common topics
- AS Maths: Pure Absolute Beginners
- AS Maths: Pure Improvers
- A level Maths: Pure Year 2
- AS & A level Maths: Statistics
- AS & A level Maths: Mechanics
- KS5 Coordinators Development

# On Demand Professional Development



## Overview

- Self-paced courses, accessed online
- Access lasts for a full year, with option to extend
- Videos, resources and support via the forums
- Free to teachers in state-funded schools or colleges in England

# ODPD available courses

| A level Maths | A level Further Maths | Core Maths | University admissions tests | Technology |
|---|---|---|---|---|
| GeoGebra in the Maths classroom | | | | |
| Desmos in the Maths Classroom | | | | |
| Numerical Methods for A level Mathematics | AS and A level Further Maths: compulsory Pure top-ups | Core Maths foundations | Preparing your students for STEP | GeoGebra in the GCSE Mathematics classroom |
| A level Maths: Statistics top-ups | A level Further Maths: Discrete, Decision and Modelling with Algorithms | Core Maths top-ups | Preparing your students for the MAT and the TMUA | |
| Imperial mA*ths project teacher support | Imperial Further mA*ths project teacher support | | | |

# Subscribe to receive our latest news

We’d love to keep you informed about our upcoming professional development courses, student enrichment events, free resources, and more!

You can subscribe to our mailing list on the AMSP website: amsp.org.uk/subscribe

# NEHS Maths Hub ’23-’24 – The Offer

| | Primary | Secondary | Post-16 | SEND |
|---|---|---|---|---|
| **Teaching for Mastery Work Groups** ***(school-wide development)*** | • Mastering Number at KS2<br>• Mastering Number at Reception and KS1<br>• Years 5-8 Continuity<br>• Oracy<br>• Readiness<br>• Development<br>• Embedding<br>• Sustaining (various strands and foci) | • Years 5-8 Continuity<br>• Developing Fluency at KS3<br>• Development<br>• Embedding Year Support<br>• Sustaining (various strands and foci) | • Supporting Low Attainers to Achieve a L2 Qualification in Mathematics<br>• Developing Core Maths Pedagogy<br>• Developing A-Level Pedagogy | • Developing Teaching for Mastery |
| **Specialist Knowledge for Teaching Mathematics Programmes** ***(individual practitioner development)*** | • Early years Teachers<br>• Primary Teachers – Number<br>• Primary Teachers – Spatial<br>• Primary ECTs Phase 1<br>• Primary ECTs Phase 2 | • Secondary ECTs Phase 1<br>• Secondary ECTs Phase 2<br>• Secondary Non-Specialists Teachers<br>• Secondary TAs | • Core Maths Teachers | |
| **Professional Learning Communities** ***(collaboration for continuous development)*** | • Mastering Number Embedding the Impact Community<br>• Strengthening Partnerships with ITT Providers Community<br>• Termly online network meetings | • Maths Subject Leaders Community<br>• Maths MAT Leaders Community<br>• Strengthening Partnerships with ITT Providers Community<br>• Termly online network meetings | • Termly online network meetings<br>• AMSP network meetings | Termly online network meetings |
| **Local Leaders of Mathematics Education Professional Development Programmes** ***(individual practitioner leadership development)*** | • Primary Mastery Specialist Programme<br>• NCETM Professional Development Lead Programme<br>• NCETM School Development Lead Programme | • Secondary Mastery Specialists Programme<br>• NCETM Professional Development Lead Programme<br>• NCETM School Development Lead Programme | • Post-16 GCSE/FS1 Mastery Specialist Programme<br>• NCETM Professional Development Lead Programme<br>• NCETM School Development Lead Programme | • NCETM PD Lead Programme<br>• NCETM SD Lead Programme |

# The AMSP/Maths Hubs Core Maths PD offer

Common Topics Subject Knowledge Live

Core Maths Top Ups On Demand PD

Core Maths Specification Networks and Pre-Release Meetings

Getting Started with Core Maths Webinar

Going to offer Core Maths

Preparing To Teach Core Maths Short Course

Start teaching Core Maths

New To Teaching Core Maths Programme

Core Maths Pedagogy Work Group

Core Maths PD Lead Programme

Core Maths Summer Festival and Core Maths Conferences

Core Maths Specialist Lead bespoke support

# Post 16 AMSP/Maths Hub offer for 2023-24 – South

Developing A level Pedagogy Work Group (15 hours of PD)

Blended

Starting from: October 2023

New to Teaching Core Maths Programme (15 hours of PD)

Online

Starting from: October 2023

Cross Phase- Supporting Low Attainers to Achieve a L2 Qualification in Mathematics Work Group (3 full days of workshops)

Developing Core Maths Pedagogy Work Group

(15 hours of PD)

Blended or Online

Starting from: November 2023

Tell us more about you and your Post 16 interests and needs. Complete our survey.

Maths Hub Post 16 Lead

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# Thank you

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